Peccei-Quinn Symmetry from Dynamical Supersymmetry Breaking

Keisuke Harigaya[a,b], Masahiro Ibe[a,b], Kai Schmitz[b], Tsutomu T. Yanagida[b]

a ICRR, The University of Tokyo, Kashiwa, Chiba 277-8582, Japan

b Kavli IPMU (WPI), UTIAS, The University of Tokyo, Kashiwa, Chiba 277-8583, Japan

Abstract

The proximity of the Peccei-Quinn scale to the scale of supersymmetry breaking in models of pure gravity mediation hints at a common dynamical origin of these two scales. To demonstrate how to make such a connection manifest, we embed the Peccei-Quinn mechanism into the vector-like model of dynamical supersymmetry breaking à la IYIT. Here, we rely on the anomaly-free discrete Z_4^R symmetry required in models of pure gravity mediation to solve the μ problem to protect the Peccei-Quinn symmetry from the dangerous effect of higher-dimensional operators. This results in a rich phenomenology featuring a QCD axion with a decay constant of $\mathcal{O}\left(10^{10}\right)$ GeV and mixed WIMP/axion dark matter. In addition, exactly five pairs of extra **5** and **5*** matter multiplets, directly coupled to the supersymmetry breaking sector and with masses close to the gravitino mass, $m_{3/2} \sim 100\,\mathrm{TeV}$, are needed to cancel the Z_4^R anomalies.

Contents

1 Introduction

1.1 Common dynamical origin of the PQ and the SUSY breaking scale

The Peccei-Quinn (PQ) mechanism [1] is widely regarded as the most promising approach to solving the strong CP problem [2] in quantum chromodynamics (QCD). Not only does it offer a dynamical explanation for the absence of CP violation in strong interactions, it also gives rise to a rich and testable phenomenology. In particular, it predicts the existence of a new pseudoscalar particle, the axion [3], which could even form dark matter (DM) or at least contribute a sizable fraction to it [4]. On the other hand, the PQ mechanism on its own is far from constituting a complete theory. At the conceptional level, it presumes the existence of an approximate global Abelian symmetry, $U(1)_{\mathrm{PQ}}$, which is spontaneously broken at some energy scale Λ_{PQ}. In order to account for the tight experimental upper bound on the QCD vacuum angle, $\bar{\theta} \lesssim 10^{-10}$ [5], this global PQ symmetry needs to be of excellent quality, which contradicts with general expectations in the context of quantum gravity [6, 7]. In fact, it is believed that in a consistent quantum theory of gravity all global symmetries are necessarily broken by gravitational interactions [8]. At low energies, this is reflected in a series of higher-dimensional, Planck-suppressed operators which explicitly introduce large symmetry-breaking effects. It is, hence, unclear what ensures that the PQ symmetry is actually of such high quality and not spoiled by gravitational effects at the Planck scale. Moreover,

the origin of the PQ scale Λ_{PQ} remains obscure. In typical axion models [9,10], the PQ scale usually takes a value close to the axion decay constant, $\Lambda_{PQ} \sim \mathcal{O}(1 \cdots 10) f_a$, where f_a is conventionally constrained (based on astrophysical [11] and cosmological observations [4]) to lie somewhere within the so-called *axion window*, $10^9 \, \text{GeV} \lesssim f_a \lesssim 10^{12} \, \text{GeV}$. The PQ symmetry is therefore likely to be broken at some intermediate scale, which appears surprising, given that one would naively expect Λ_{PQ} to be either tied to the electroweak scale [1,3], $\Lambda_{PQ} \sim 10^2 \, \text{GeV}$, or, in absence of any other scale, to the string or Planck scale, $\Lambda_{PQ} \sim \left(10^{17} \cdots 10^{18} \, \text{GeV}\right) / \left(32\pi^2\right) \sim 10^{15} \cdots 10^{16} \, \text{GeV}$ [12]. The first of these two possibilities has, however, been ruled out experimentally a long time ago [13], while the second one requires fine-tuning of the initial axion misalignment angle to avoid the overproduction of axionic DM [14]. This leaves one wondering what other new physics might possibly come into question as the dynamical origin of the scale Λ_{PQ}.

The above mentioned problems related to the PQ mechanism appear in a new light as soon as supersymmetry (SUSY) is brought into play. Just like the PQ symmetry, SUSY needs to be spontaneously broken at some intermediate energy scale, Λ_{SUSY}. This entails the tempting idea to suppose that Λ_{PQ} and Λ_{SUSY} are, in fact, determined by the same dynamics [15,16]. For instance, we may imagine that SUSY is broken dynamically by the interactions in some strongly coupled hidden sector. The virtue of such models of dynamical SUSY breaking (DSB) [17] is that, in these models, the SUSY breaking scale is generated without any dimensionful input parameters via the effect of dimensional transmutation, i.e., it ends up being related to the dynamical scale Λ of the strong interactions, $\Lambda_{SUSY} \sim \Lambda$. If the same strong dynamics are responsible for the spontaneous breaking of the PQ symmetry, we then have

$$\Lambda_{SUSY} \sim \Lambda \sim \Lambda_{PQ} \sim 10^{11} \cdots 10^{12} \, \text{GeV} \,. \tag{1}$$

Remarkably enough, this estimate coincides with the range of Λ_{SUSY} values that one typically encounters in models of *pure gravity mediation* (PGM) [18,19] (for closely related mediation schemes, see [20,21]). In this framework for the mediation of SUSY breaking to the visible sector, sfermions receive large masses of the order of the gravitino mass $m_{3/2} \sim 10^4 \cdots 10^6 \, \text{GeV}$ via the tree-level scalar potential in supergravity (SUGRA) [22], while gauginos obtain one loop-suppressed masses via anomaly mediation [23] (see also [24]). PGM comes with a number of attractive features at the cost of a slightly fine-tuned electroweak scale: It easily accounts for a standard model (SM) Higgs boson mass of $126 \, \text{GeV}$ [25] thanks to large stop loop corrections [26]; it ensures the unification of the SM gauge couplings, so that it may be readily embedded into a grand unified theory (GUT); and it is capable of providing a viable candidate for dark matter either in the form of winos [19, 21, 27], binos [28] or higgsinos [29]. At the same time, it avoids the Polonyi problem usually encountered in models of gravity mediation [30], as SUSY is required to be broken by a non-singlet field in PGM; it is free of any cosmological gravitino problem [31], as the gravitino decays way before the onset of big bang nucleosynthesis; and it is in less tension with constraints on flavor-changing neutral currents (FCNCs) and CP violation [32], once again because of the high sfermion mass scale.

1.2 Unique discrete R symmetry for pure gravity mediation

Meanwhile, a common idea to protect the PQ symmetry from the dangerous effects of higher-dimensional operators is to invoke some gauge symmetry which ensures that the PQ symmetry survives as an approximate *accidental* symmetry in the low-energy effective theory [7, 33, 34]. A particularly attractive choice in this context, which we have recently examined in more detail in [34], is to protect the PQ symmetry by means of a gauged discrete R symmetry, Z_N^R. Such a discrete R symmetry may be the remnant of a spontaneously broken continuous symmetry in higher dimensions (for instance, in string theory) [35]; and it is an often important and sometimes even imperative ingredient in SUSY model building: It prevents too rapid proton decay via perilous dimension-5 operators [36]; it forbids a constant term in the superpotential of the order of the Planck scale, which would otherwise result in a huge negative cosmological constant [37]; and it may account for the approximate global continuous R symmetry which is required to realize stable [38] or meta-stable [39] SUSY-breaking vacua in a large class of DSB models.

On top of that, a discrete R symmetry automatically suppresses the bilinear Higgs mass in the superpotential, $W \supset \mu H_u H_d$, and hence allows for a solution to the μ problem [40] in the minimal supersymmetric standard model (MSSM). In fact, in absence of a bare μ term, a supersymmetric mass for the MSSM Higgs fields H_u and H_d needs to be generated in consequence of R symmetry breaking. This can, for instance, be done as in the next-to-minimal supersymmetric standard model (NMSSM) [41], where one introduces a SM singlet field S that couples to $H_u H_d$ and which obtains a vacuum expectation value (VEV) $\langle S \rangle = \mu$ during electroweak symmetry breaking (EWSB). A more minimal and hence more elegant solution, however, consists in allowing for a Higgs bilinear term [42] in the Kähler potential (see also [43]),

$$K \supset c_H H_u H_d \,, \tag{2}$$

where c_H is a dimensionless constant of $\mathcal{O}(1)$ and which readily yields a μ term of the order of the gravitino mass, $\mu = c_H \, m_{3/2}$, after R symmetry breaking. This solution to the μ problem is typically employed in PGM models, where it renders the μ and B_μ parameters in the MSSM Higgs potential linearly independent, thereby ensuring the successful occurrence of radiative EWSB [44]. On the other hand, the term in Eq. (2) is only allowed in the Kähler potential as long as the total R charge of the Higgs bilinear $H_u H_d$ is zero modulo N. In order to assess for which Z_N^R symmetries this requirement can be fulfilled, it is important to remember that any discrete symmetry which is supposed to be relevant at low energies should be a good gauge symmetry even at the quantum level, i.e., it should be free of any gauge anomalies [45, 46]. This implies in particular as a minimal constraint that the fermions of the MSSM should equally contribute to the weak and color anomalies of the discrete R symmetry,[1] $\mathcal{A} \left[Z_N^R - SU(2)_L - SU(2)_L \right] - \mathcal{A} \left[Z_N^R - SU(3)_C - SU(3)_C \right] = 0$. For generation-independent R charges commuting with $SU(5)$ and together with the structure of

---

[1]Extra matter fields contributing to these two anomalies should appear in complete $SU(5)$ multiplets, so as not to disturb the unification of the SM gauge couplings. As such, they, too, should equally contribute to the weak and color anomalies of discrete R symmetry, leaving the difference between these two anomalies unaffected [47].

the MSSM superpotential (supplemented by heavy neutrino Majorana mass terms in accord with the seesaw mechanism [48]), this condition then necessitates that [47] (see also [34, 49])

$$r_{H_u} + r_{H_d} \stackrel{(N)}{=} 4 \,. \tag{3}$$

Here, we have introduced r_X as the general symbol to denote the R charge of the field X and where $\stackrel{(N)}{=}$ serves as a shorthand notation for equality modulo N. Consequently, only for $N = 4$, the R charge of the Higgs bilinear vanishes,[2] which singles out Z_4^R as the only discrete R symmetry consistent with PGM and the generation of the μ term via the Higgs Kähler term in Eq. (2).

1.3 Synopsis: PQ symmetry from dynamical SUSY breaking

Together, the above observations lead to a remarkably consistent picture in the context of pure gravity mediation: *While the magnitude of the PQ scale may be determined by the SUSY-breaking scale in some appropriate DSB model, the quality of the PQ symmetry may be safeguarded by the discrete Z_4^R symmetry required to solve the μ problem.* In this paper, we shall demonstrate that such a scenario is indeed feasible—we set out to construct an explicit DSB model featuring an approximate PQ symmetry that is sufficiently protected by an anomaly-free discrete Z_4^R symmetry. In doing so, we will restrict ourselves to the arguably simplest case, i.e., we will content ourselves with presenting a minimal example based on strong $SU(2)$ dynamics breaking SUSY à la IYIT [51]. Furthermore, to render the Z_4^R symmetry anomaly-free, we are lead to introduce a set of new $SU(5)$ multiplets,[3] which obtain masses via their coupling to the SUSY-breaking sector. Here, the phenomenological constraints on our model surprisingly single out a unique number of extra matter fields: five pairs of **5** and **5*** multiplets with supersymmetric masses close to $m_{3/2}$. Such additional matter states affect the gaugino mass spectrum in PGM [28, 54] and thus play a crucial role in determining the composition of the lightest supersymmetric particle (LSP). This has, in turn, important consequences for dark matter and SUSY searches at colliders. At the same time, for gravitino masses around $100 \, \text{TeV}$, our model predicts an axion decay constant f_a of $\mathcal{O}\left(10^{10}\right)$ GeV and an axionic contribution to the total abundance of dark matter of at most $\mathcal{O}\left(10\,\%\right)$. Meanwhile, the superpartners of the axion, the axino as well as the saxion, receive masses of the order of the SUSY breaking scale and hence do not cause any cosmological problems [16]. We therefore anticipate our model to offer an appealing solution to the strong CP problem in the context of dynamical SUSY breaking, which is consistent with all existing bounds, but which, at the same time, can be readily probed in a number of terrestrial experiments and astrophysical observations.

The rest of the paper is organized as follows: In the next section, we will show how to embed the PQ mechanism into the IYIT model of dynamical SUSY breaking. Here, some of the technical details regarding the vacuum configuration of the IYIT model after taking into account R symmetry

---

[2]Note that Z_2^R is not an actual R symmetry; instead it is equivalent to the non-R matter parity of the MSSM [50].

[3]We do not consider the possibility of anomaly cancellation via the Green-Schwarz mechanism in string theory [52]. In such a case, the Z_4^R (and several other discrete R symmetries) could also be rendered anomaly-free solely within the MSSM, i.e., without the need for an extra matter sector [53].

breaking have been deferred to Appendix A. In Sec. 3, we will then comment on the quality of the PQ symmetry in dependence of the free parameters of our model and discuss the resulting phenomenological constraints. Finally, we will conclude and give a brief outlook in Sec. 4.

2 Embedding of the PQ mechanism into the IYIT model

2.1 Field content and superpotential of the SUSY-breaking sector

The IYIT model represents the simplest vector-like model of dynamical SUSY breaking. In its most general formulation, it is based on strongly interacting $Sp(N)$ gauge dynamics, while featuring $2N_f = 2(N+1)$ matter fields Ψ^i in the fundamental representation of $Sp(N)$. At energies below the dynamical scale Λ, this theory exhibits a flat quantum moduli space, which is best described in terms of the $N_f(2N_f - 1)$ gauge-invariant composite "meson" fields

$$M^{ij} \simeq \frac{1}{\eta}\frac{1}{\Lambda}\left\langle \Psi^i \Psi^j \right\rangle, \quad i, j = 1, 2, \cdots 2N_f. \tag{4}$$

Here, η is a dimensionless coefficient which is supposed to ensure that the meson fields M^{ij} are canonically normalized at low energies. According to arguments from *naive dimensional analysis* (NDA) [55], it is expected to be of $\mathcal{O}(4\pi)$; but values as small as $\eta \simeq \pi$ may perhaps also still be admissible. Similar numerical coefficients are expected to appear in a number of places in the low-energy description of the IYIT model. For simplicity, we will, however, ignore the possibility that these factors could numerically deviate from each other and simply account for the uncertainties in all dimensionless couplings of our model by means of a *single* NDA factor η.

As shown by Seiberg, the meson fields in Eq. (4) are subject to the following quantum mechanically deformed moduli constraint [56],

$$\mathrm{Pf}\left(M^{ij}\right) \simeq \left(\frac{\Lambda}{\eta}\right)^{N+1}, \tag{5}$$

where the dynamical scale on the right-hand side of this constraint arises in consequence of non-perturbative effects and where $\mathrm{Pf}(M)$ denotes the Pfaffian of the antisymmetric meson matrix M, $[\mathrm{Pf}(M)]^2 = \det(M)$. A convenient way to implement this constraint when studying the low-energy dynamics of the IYIT model is to rewrite it in the form of a dynamical superpotential,

$$W_{\mathrm{dyn}} \simeq \kappa\,\eta\,X\left(\frac{\eta}{\Lambda}\right)^{N-1}\left[\mathrm{Pf}\left(M^{ij}\right) - \left(\frac{\Lambda}{\eta}\right)^{N+1}\right], \tag{6}$$

with the field X acting as a Lagrange multiplier. Unfortunately, the dynamical Kähler potential for X is uncalculable and hence the physical interpretation of X is ambiguous. One possibility to account for this ambiguity is to *define* the dimensionless coupling κ such that X is always canonically normalized, irrespective of its physical status. More precisely, if a dynamical Kähler potential for X should be generated, X would be physical and we would expect κ to be some $\mathcal{O}(1)$ constant. On the other hand, if X should remain unphysical, we could still introduce X as in

Eq. (6), the only difference being that, now, κ would formally blow up to infinity. In what follows, we will therefore stick to the dynamical superpotential in Eq. (6), keeping in mind that there are two sensible regimes for the coupling κ: We should either set $\kappa \sim 1$ or take the limit $\kappa \to \infty$.

Supersymmetry is broken in the IYIT model by lifting the flat directions in moduli space by means of appropriate Yukawa couplings. Let us introduce a singlet field Z_{ij} for each M^{ij} and couple these singlets to the corresponding "quark" bilinears in the tree-level superpotential,

$$W_{\text{tree}}^{\text{IYIT}} = \frac{1}{2}\lambda'_{ij}\, Z_{ij}\, \Psi^i \Psi^j \,. \tag{7}$$

At low energies, these Yukawa interactions then give rise to the following effective superpotential,

$$W_{\text{eff}}^{\text{IYIT}} \simeq \frac{1}{2}\lambda_{ij}\, \frac{\Lambda}{\eta}\, Z_{ij}\, M^{ij} \,, \tag{8}$$

where we have replaced the high-energies Yukawa couplings λ'_{ij} by their low-energy analogues λ_{ij} to account for their RGE running from energies high above the dynamical scale to energies below the dynamical scale. For all Yukawa couplings λ_{ij} being nonzero, the superpotential in Eq. (8) implies F-term conditions for the meson fields, $M = 0$, which contradict the deformed moduli constraint in Eq. (5), $\text{Pf}(M) \neq 0$, signaling that SUSY is spontaneously broken. Another way to put this is to say that the total effective superpotential at low energies,

$$W_{\text{eff}} = W_{\text{dyn}} + W_{\text{eff}}^{\text{IYIT}} \simeq \kappa\,\eta\, X \left(\frac{\eta}{\Lambda}\right)^{N-1} \left[\text{Pf}\left(M^{ij}\right) - \left(\frac{\Lambda}{\eta}\right)^{N+1}\right] + \frac{1}{2}\lambda_{ij}\, \frac{\Lambda}{\eta}\, Z_{ij}\, M^{ij} \,, \tag{9}$$

is of the ORaifeartaigh type and, hence, SUSY is broken via the ORaifeartaigh mechanism [57].

An interesting feature of the IYIT model, which will be of crucial importance to us in the following, is that the superpotential in Eq. (7) is invariant under an axial $U(1)_A$ symmetry associated with a global Ψ^i phase rotation, $\Psi^i \to \exp\left(iq_i\theta\right)\Psi^i$, which is anomaly-free under the strongly coupled $Sp(N)$ gauge group. In [58], Domcke et al. have promoted this $U(1)_A$ to an exact gauge symmetry, $U(1)_A \to U(1)_{\text{FI}}$, to point out a possibility how to dynamically generate a field-dependent Fayet-Iliopoulos (FI) D-term in field theory. In this paper, we will now identify the same global $U(1)_A$ as the PQ symmetry, $U(1)_A \to U(1)_{\text{PQ}}$, and show that it may very well serve as a basis for the construction of a viable axion model.

2.2 SUSY- and PQ symmetry-breaking vacuum at low energies

In the remainder of this paper, we shall focus on the simplest version of the IYIT model, i.e., SUSY breaking via strong $Sp(1) \cong SU(2)$ dynamics. The extension of our construction to the general $Sp(N)$ case is straightforward; it merely requires a bigger notational effort. For $N = 1$, we then have to deal with $N_f = 2$ quark flavors, each consisting of a pair of fundamental quarks fields, $\left(\Psi^1, \Psi^2\right)$ and $\left(\Psi^3, \Psi^4\right)$. Under the $U(1)_A$ flavor symmetry, these fields are charged as follows,

$$\left[\Psi^1\right] = \left[\Psi^2\right] = +\frac{1}{2}\,, \quad \left[\Psi^3\right] = \left[\Psi^4\right] = -\frac{1}{2}\,, \tag{10}$$

where we have chosen the normalization so that the mesons at low energies carry integer charges. In fact, relabeling all meson and singlet fields, M^{ij} and Z_{ij}, according to their $U(1)_A$ charges, the low-energy effective theory ends up consisting of the following degrees of freedom (DOFs),

$$M_+ = M^{12}, \quad M_- = M^{34}, \quad M_0^1 = M^{13}, \quad M_0^2 = M^{14}, \quad M_0^3 = M^{23}, \quad M_0^4 = M^{24}, \quad (11)$$

$$Z_- = Z_{12}, \quad Z_+ = Z_{34}, \quad Z_0^1 = Z_{13}, \quad Z_0^2 = Z_{14}, \quad Z_0^3 = Z_{23}, \quad Z_0^4 = Z_{24}.$$

In terms of these charge eigenstates, the effective superpotential in Eq. (9) now reads

$$W_{\text{eff}} \simeq \kappa\,\eta\,X\left[\text{Pf}\left(M^{ij}\right) - \left(\frac{\Lambda}{\eta}\right)^2\right] + \frac{\Lambda}{\eta}\left(\lambda_+ M_+ Z_- + \lambda_- M_- Z_+ + \lambda_0^a M_0^a Z_0^a\right), \quad (12)$$

with $a = 1, 2, 3, 4$, where we have renamed the Yukawa couplings λ_{ij} in an obvious manner and where the Pfaffian of the meson matrix can now be expanded into the following polynomial,

$$\text{Pf}\left(M^{ij}\right) = M_+ M_- - M_0^1 M_0^4 + M_0^2 M_0^3. \quad (13)$$

The F-term scalar potential corresponding to this superpotential exhibits a saddle point at the origin as well as three local minima, in which the Pfaffian constraint is either approximately satisfied by the meson bilinear $M_+ M_-$, by $M_0^1 M_0^4$ or by $M_0^2 M_0^3$, with all other meson VEVs vanishing. The potential energies of these three vacua respectively scale with the products of the corresponding Yukawa couplings, $\lambda_+ \lambda_-$, $\lambda_0^1 \lambda_0^4$, and $\lambda_0^2 \lambda_0^3$. As we intend to identify the $U(1)_A$ flavor symmetry with the PQ symmetry, we need to make sure that the $U(1)_A$ symmetry is spontaneously broken at low energies. This is, however, only achieved once the charged mesons M_+ and M_- acquire nonzero VEVs. In the following, we shall therefore assume that the product $\lambda_+ \lambda_-$ is (at least slightly) smaller than $\lambda_0^1 \lambda_0^4$ and $\lambda_0^2 \lambda_0^3$, so that $M_+ M_- \sim \Lambda^2/\eta^2$ and $M_0^a = Z_0^a = 0$ in the true vacuum. Setting all neutral mesons and singlets to zero in Eq. (12), we then obtain the effective superpotential describing the fluctuations of M_+ and M_- around the symmetry-breaking vacuum,

$$W_{\text{eff}} \simeq \kappa\,\eta\,X\left[M_+ M_- - \left(\frac{\Lambda}{\eta}\right)^2\right] + \frac{\Lambda}{\eta}\left(\lambda_+ M_+ Z_- + \lambda_- M_- Z_+\right). \quad (14)$$

In the limit of rigid SUSY, Z_+, Z_- and X turn out to be stabilized at zero (see Appendix A). The VEVs of the meson fields M_+ and M_- then readily follow from minimizing the F-term scalar potential corresponding to the above superpotential. For canonical Kähler potential, we obtain

$$M_\pm = \varepsilon\,\frac{\lambda}{\lambda_\pm}\frac{\Lambda}{\eta}, \quad \lambda = \sqrt{\lambda_+ \lambda_-}, \quad \varepsilon = (1 - \zeta)^{1/2}, \quad \zeta = \left(\frac{\lambda}{\kappa\eta}\right)^2, \quad (15)$$

where λ denotes the positive square root of the geometric mean of λ_+^2 and λ_-^2 and with ε parameterizing the suppression of $M_\pm$ w.r.t. the asymptotic expression in the limit $\kappa \to \infty$ (or $\zeta \to 0$),

$$M_\pm = \varepsilon\,M_\pm^0, \quad M_\pm^0 = \lim_{\zeta \to 0} M_\pm = \frac{\lambda}{\lambda_\pm}\frac{\Lambda}{\eta}. \quad (16)$$

Here, ζ can be interpreted as a measure for the coupling strength of the Yukawa terms in Eq. (14), viz. λ, in comparison to the coupling strength of the Lagrange term, viz. $\kappa\eta$, which arises in

consequence of the deformed moduli constraint. For fixed values of κ and η, unitarity and the fact that ζ must not exceed 1 (so that the meson VEVs do not vanish and the PQ symmetry is, in fact, broken) then restrict the Yukawa coupling λ to take a value between 0 and $\lambda_{\max} \simeq \min\{4\pi, \kappa\,\eta\}$. Consequently, the parameters λ, ζ and ε are allowed to vary within the following ranges,

$$0 \leq \lambda \leq \lambda_{\max}\,, \quad 0 \leq \zeta \leq \zeta_{\max} = \left(\frac{\lambda_{\max}}{\kappa\,\eta}\right)^2\,, \quad (1 - \zeta_{\max})^{1/2} = \varepsilon_{\min} \leq \varepsilon \leq 1\,. \tag{17}$$

It is important to keep in mind that the above results for the meson VEVs come with a certain irreducible uncertainty, given the fact that we are unable to calculate the precise form of the Kähler potential below the dynamical scale. In this sense, the exact parameter relations in Eqs. (15), (16) and (17) should be taken with a grain of salt, as they may easily receive corrections from the noncanonical Kähler potential. On the other hand, we will continue to use the above relations in the following analysis *for definiteness*. After all, they allow for a *consistent* treatment of our model in terms of a well-defined set of parameters with a clear physical interpretation. In other words, in the following, we will study the IYIT model for the special case of a canonical Kähler potential, guided by the notation that this should represent an important *benchmark scenario for the more general case*—a benchmark scenario that we have well under control. This will, in particular, help us to keep track of the various numerical factors in our analysis and provide us with better estimates for a handful of prefactors which one would otherwise simply take to be "of $\mathcal{O}(1)$".

Plugging our result for the meson VEVs in Eq. (15) back into the superpotential in Eq. (14), we are able to deduce the SUSY-breaking F-terms of the fields Z_+, Z_- and X,

$$\left|F_{Z_\pm}\right| = \lambda\,(1 - \zeta)^{1/2}\,\frac{\Lambda^2}{\eta^2} = \zeta^{1/2}\,(1 - \zeta)^{1/2}\,F_0\,, \quad \left|F_X\right| = \frac{\lambda^2}{\kappa\,\eta}\,\frac{\Lambda^2}{\eta^2} = \zeta\,F_0\,, \quad F_0 = \kappa\,\eta\,\frac{\Lambda^2}{\eta^2}\,. \tag{18}$$

The parameter ζ can hence also be regarded as a measure for the size of the F-term of the field X,

$$\zeta = \frac{\left|F_X\right|}{F_0} \leq 1\,. \tag{19}$$

Here, the fact that F_X can be nonzero in the first place indicates (perhaps somewhat surprisingly) that, for $\zeta > 0$, the deformed moduli constraint is actually not exactly satisfied in the true vacuum,

$$\left|M_+ M_- - \left(\frac{\Lambda}{\eta}\right)^2\right| = \zeta\left(\frac{\Lambda}{\eta}\right)^2\,, \tag{20}$$

where ζ serves again as a useful measure to parametrize the deviation from the situation in the limit $\kappa \to \infty$ (where the moduli constraint *is* exactly fulfilled in the true vacuum). Moreover, given the possibility of nonzero F_X, the field X may turn out to be the most important one among the three SUSY-breaking fields Z_+, Z_- and X. According to Eq. (18), we namely have

$$\left|\frac{F_X}{F_{Z_\pm}}\right| = \left(\frac{\zeta}{1 - \zeta}\right)^{1/2}\,, \tag{21}$$

so that $|F_X| < \left|F_{Z_\pm}\right|$ for $\zeta < 1/2$ and $|F_X| \geq \left|F_{Z_\pm}\right|$ for $\zeta \geq 1/2$ in a nicely symmetric fashion. At this point, we should mention that the authors of [58] exclusively focused on the regime of

large κ and hence small ζ, thereby disregarding the possibility that the meson VEVs in Eq. (16) could potentially be suppressed. In this paper, we shall, by contrast, refrain from making any such assumption regarding the size of ζ and simply keep ζ as a free parameter in our analysis. This will allow us to consistently account for the possibility of suppressed meson VEVs, $M_\pm = \varepsilon\, M_\pm^0$, relative to the naive expressions $M_\pm^0$ which we expect in the limit $\kappa \to \infty$. We emphasize that, adapting this procedure, we are not only able to capture a possible suppression of the meson VEVs due to a nonvanishing F-term for the field X, but—at an effective, phenomenological level—also a possible suppression due to the uncalculable strong-coupling effects in the Kähler potential. This is one of the main reasons why we decide to stick to our parametrization in terms of ζ in the following, despite the uncertainties induced by the unknown terms in the dynamical Kähler potential.

Our results for the singlet F-terms in Eq. (18) now immediately provide us with an expression for the SUSY breaking scale $\Lambda_{\mathrm{SUSY}} \equiv \mu$,

$$\Lambda_{\mathrm{SUSY}}^2 \equiv \mu^2 = \left(\left|F_{Z_+}\right|^2 + \left|F_{Z_-}\right|^2 + \left|F_X\right|^2 \right)^{1/2} = \lambda\,(2-\zeta)^{1/2}\,\frac{\Lambda^2}{\eta^2}\,, \tag{22}$$

which leads us to yet another interpretation of the parameter ζ. From Eq. (22), we infer that $2-\zeta$ counts what may be regarded as the effective number of "active" SUSY-breaking fields $N_{\mathrm{SUSY}}^{\mathrm{eff}}$, so that $\zeta = 2 - N_{\mathrm{SUSY}}^{\mathrm{eff}}$. More precisely, for $\zeta = 0$, the F-term of the field X vanishes and SUSY is solely broken by the F-terms belonging to the fields Z_+ and Z_-. Hence, $N_{\mathrm{SUSY}}^{\mathrm{eff}} = 2$ in this case. On the other hand, for $\zeta = 1$, both F_{Z_+} and F_{Z_-} are zero. SUSY is then solely broken by the F-term of the field X and $N_{\mathrm{SUSY}}^{\mathrm{eff}} = 1$. Correspondingly, intermediate values of ζ interpolate between these two extrema of $N_{\mathrm{SUSY}}^{\mathrm{eff}}$. Furthermore, we are now able to determine the gravitino mass $m_{3/2}$, which appears as a constant term, $W_0 = \mathrm{const.}$, in the superpotential upon R symmetry breaking,[4]

$$W \supset W_0 = m_{3/2}\, M_{\mathrm{Pl}}^2\,, \tag{23}$$

with $M_{\mathrm{Pl}} = (8\pi G)^{-1/2} \simeq 2.44 \times 10^{18}\,\mathrm{GeV}$ denoting the reduced Planck mass. Requiring that the true vacuum at low energies be a Minkowski vacuum with (almost) zero cosmological constant, we then have to balance the SUSY-breaking contribution to the full SUGRA scalar potential, $\Lambda_{\mathrm{SUSY}}^4$, against the constant term, $-3/M_{\mathrm{Pl}}^2\,|W_0|^2$, induced by R symmetry breaking. This gives

$$m_{3/2} = \frac{\Lambda_{\mathrm{SUSY}}^2}{\sqrt{3}\, M_{\mathrm{Pl}}} = \frac{\lambda\,(2-\zeta)^{1/2}}{\sqrt{3}\,\eta^2}\,\frac{\Lambda^2}{M_{\mathrm{Pl}}}\,. \tag{24}$$

In conclusion, we find that, as anticipated in the introduction, both the SUSY breaking scale as well as the gravitino mass turn out to be controlled by the dynamical scale Λ. In order to attain a gravitino mass consistent with PGM, say, $m_{3/2} \sim 100\,\mathrm{TeV}$, we thus need Λ to be of $\mathcal{O}\left(10^{12}\right)\,\mathrm{GeV}$. In the next section, we shall see how this scale can also be understood as the scale of PQ symmetry breaking and, in particular, how it is related to the axion decay constant f_a.

---

[4]Here, we ignore the VEV of the Kähler potential, K_0, which actually enters the right-hand side of this relation in form of a factor $\exp\left(-K_0/M_{\mathrm{Pl}}^2/2\right)$. Since $K_0 \ll M_{\mathrm{Pl}}^2$ (see Eq. (27)), this factor is, however, completely negligible.

2.3 Identification and decay constant of the axion

The nonzero VEVs of the charged meson fields M_+ and M_- in Eq. (15) spontaneously break the global $U(1)_A$ symmetry of the IYIT superpotential. We identify this flavor symmetry with the PQ symmetry, which means that the chiral axion superfield A must correspond to the goldstone multiplet of spontaneous $U(1)_A$ breaking contained in M_+ and M_-. To make this relation manifest, let us expand M_+ and M_- around their VEVs (which satisfy $\lambda_+ \langle M_+ \rangle = \lambda_- \langle M_- \rangle$) into a flavor-symmetric fluctuation M in the radial direction as well as into a goldstone phase Θ,

$$M_\pm = \frac{1}{\lambda_\pm} \left[\lambda_\pm \langle M_\pm \rangle + \frac{\lambda_h}{\sqrt{2}} M \right] e^{\pm \Theta} . \tag{25}$$

Here, the coupling λ_h, denoting the positive square root of the harmonic mean of λ_+^2 and λ_-^2,

$$\lambda_h = \left[\frac{1}{2} \left(\frac{1}{\lambda_+^2} + \frac{1}{\lambda_-^2} \right) \right]^{-1/2} , \tag{26}$$

is chosen such that the meson field M, i.e., the "Higgs field" of PQ symmetry breaking, is canonically normalized. Meanwhile, the phase Θ directly corresponds to the axion multiplet A modulo a proper normalization. In order to find the precise relation between Θ and A, we have to examine the canonical Kähler potential for M_+ and M_- in the true vacuum, i.e., for $M = 0$,

$$K = K_0 \cosh \left(\Theta + \Theta^\dagger \right) + \Delta \sinh \left(\Theta + \Theta^\dagger \right) = K_0 + \Delta \left(\Theta + \Theta^\dagger \right) + \frac{1}{2} K_0 \left(\Theta + \Theta^\dagger \right)^2 + \cdots , \tag{27}$$

where K_0 stands for the VEV of the mesonic Kähler potential and Δ denotes the difference between the two meson VEVs squared,[5]

$$K_0 = \left\langle |M_+|^2 \right\rangle + \left\langle |M_+|^2 \right\rangle , \quad \Delta = \left\langle |M_+|^2 \right\rangle - \left\langle |M_+|^2 \right\rangle . \tag{28}$$

This leads us to identify the properly normalized axion field A as follows,

$$A = K_0^{1/2} \Theta , \quad K_0 = \frac{2}{\rho^2} (1 - \zeta) \left(\frac{\Lambda}{\eta} \right)^2 , \quad \rho = \frac{\lambda_h}{\lambda} = \left[\frac{1}{2} \left(\frac{\lambda_+}{\lambda_-} + \frac{\lambda_-}{\lambda_+} \right) \right]^{-1/2} , \tag{29}$$

Here, the complex scalar $(\phi + i\, a)/\sqrt{2}$ contained in $A = \{\phi, a, \tilde{a}\}$ now consists of the axion a and the saxion ϕ, while the fermionic component of A represents the axino $\tilde{a}$. Meanwhile, $\rho \in [0, 1]$ is a convenient measure for the magnitude of the flavor hierarchy in Eq. (14). For equal Yukawa couplings, $\lambda_+ = \lambda_-$, the superpotential in Eq. (14) is invariant under the exchange of "+" and "−" and $\rho = 1$. On the other hand, as soon as $\lambda_+ \neq \lambda_-$, this symmetry is broken and $\rho < 1$.[6] The more

---

[5]Note that Δ would represent a field-dependent and dynamically generated FI-term for the $U(1)_A$ flavor symmetry, in case this symmetry was promoted to a gauge symmetry and the meson VEVs were not identical, $\lambda_+ \neq \lambda_-$ [58].

[6]At high energies and for generic "neutral" Yukawa couplings λ_0^a, the exchange symmetry of the superpotential in Eq. (14) for $\lambda_+ = \lambda_-$ cannot be realized at the level of the fundamental quarks Ψ^i. This renders it an accidental rather than an exact symmetry, which is expected to be explicitly broken in the Kähler potential. More generally, we stress that, out of the maximal flavor symmetry of the IYIT model, $U(4)$, we only depend on the PQ symmetry, $U(4) \supset SU(4) \supset U(1)_{\mathrm{PQ}}$, to be a reasonably good symmetry. All other global symmetries ought to be explicitly broken, in order to avoid massless particles and/or the formation of topological defects in the early universe.

we increase the flavor hierarchy in the charged meson sector, the smaller ρ then becomes—until, for $\lambda_+ \gg \lambda_-$ or $\lambda_+ \ll \lambda_-$, it eventually approaches zero. However, for not too large a hierarchy, $|\log_{10}(\lambda_+/\lambda)| \leq 1/2$, the parameter ρ always stays rather close to unity, $\rho \geq (20/101)^{1/2} \simeq 0.44$. We will therefore ignore the possibility of a parametrically suppressed value of ρ in the following and simply take it to be some $\mathcal{O}(1)$ constant from now on, i.e., we will work with $\rho \sim 0.3 \cdots 1$.

Next, let us determine the axion decay constant f_a. As we will discuss in Sec. 3.1, the PQ symmetry ends up acquiring a color anomaly due to the presence of additional matter states coupling directly to the SUSY-breaking sector. This is good news, since a PQ color anomaly is a necessary prerequisite for any implementation of the PQ mechanism; it generates the $a\, G\tilde{G}$ term by means of which the CP-violating $\bar{\theta}$ term in the effective QCD Lagrangian is eventually canceled,

$$\mathcal{L}_{\text{eff}}^{\text{QCD}} \supset \bar{\theta}\, \frac{\alpha_s}{8\pi}\, \text{Tr}\left[G_{\mu\nu}\, \tilde{G}^{\mu\nu}\right] - |\mathcal{A}_{\text{PQ}}|\, \frac{a/\sqrt{2}}{K_0^{1/2}}\, \frac{\alpha_s}{8\pi}\, \text{Tr}\left[G_{\mu\nu}\, \tilde{G}^{\mu\nu}\right] . \tag{30}$$

Here, α_s is the strong coupling constant, $G_{\mu\nu}$ and $\tilde{G}^{\mu\nu}$ respectively denote the gluon field strength tensor and its dual and $\mathcal{A}_{\text{PQ}}$ stands for the coefficient of the $U(1)_{\text{PQ}}$–$SU(3)_C$–$SU(3)_C$ anomaly. The axion decay constant f_a is now *defined* such that these two terms can be combined to yield

$$\mathcal{L}_{\text{eff}}^{\text{QCD}} \supset \left(\bar{\theta} - \frac{a}{f_a}\right) \frac{\alpha_s}{8\pi}\, \text{Tr}\left[G_{\mu\nu}\, \tilde{G}^{\mu\nu}\right] , \tag{31}$$

which tells us that the decay constant f_a is basically given by the VEV of the Kähler potential,

$$f_a = \frac{\sqrt{2}}{|\mathcal{A}_{\text{PQ}}|}\, K_0^{1/2} = \frac{2}{|\mathcal{A}_{\text{PQ}}|}\, \frac{1}{\rho}\, (1 - \zeta)^{1/2}\, \frac{\Lambda}{\eta} = \frac{2}{|\mathcal{A}_{\text{PQ}}|}\, \frac{\varepsilon}{\rho}\, \frac{\Lambda}{\eta} . \tag{32}$$

In view of this result, three comments are in order: (i) Irrespectively of the details of how the PQ mechanism is actually implemented in a concrete model, naive dimensional analysis leads us to expect that f_a should be suppressed compared to the PQ breaking scale by one power of the NDA parameter η [55]. Our result in Eq. (32) obviously complies with this expectation. (ii) In addition to this, the axion decay constant turns out to be further suppressed due to various factors beyond the simple NDA estimate, $f_a \sim \Lambda/\eta$. It is also suppressed by the anomaly coefficient $|\mathcal{A}_{\text{PQ}}|$ as well as by the suppression factor ε in the meson VEVs. Depending on the size of these prefactors, f_a may be smaller than the dynamical scale Λ by one or even more orders of magnitude. For $\varepsilon \sim 0.1$, $|\mathcal{A}_{\text{PQ}}| \sim 1 \cdots 10$ and $\eta \sim \pi \cdots 4\pi$, for instance, we typically have $f_a \sim 10^{-2}\, \Lambda$, so that SUSY breaking scales of $\mathcal{O}\left(10^{12}\right)$ GeV result in axion decay constants of $\mathcal{O}\left(10^{10}\right)$ GeV. In fact, as we will see in Secs. 3.2 and 3.3, the Λ and f_a values consistent with all phenomenological constraints will happen to be of exactly these orders of magnitude. (iii) Similarly to the meson VEVs in Eq. (15), the relation between f_a and Λ in Eq. (32) is sensitive to corrections coming from the dynamically generated Kähler potential. Imagine, for instance, that the mesons $M_\pm$ obtain a quartic Kähler potential due to strong-coupling effects,

$$K = M_\pm M_\pm^\dagger + \Delta K \equiv (1 + c_\pm)\, M_\pm M_\pm^\dagger , \quad \Delta K = \pm C_\pm \left(\frac{\eta}{\Lambda}\right)^2 \left(M_\pm M_\pm^\dagger\right)^2 , \tag{33}$$

where the size of the coefficients $C_\pm$ is unknown and where the $c_\pm = \Delta K / (M_\pm M_\pm^\dagger)$ parametrize the ratio between the noncanonical corrections and the canonical Kähler potential. Large corrections, $|c_\pm| \gg 0$, therefore shift the normalization of the meson fields as well as the VEV of the Kähler potential, K_0, which in turn modifies the relation between f_a and Λ in Eq. (32). Such large corrections do, however, not drastically affect our final conclusions, as all bounds that we are going to study in Sec. 3.2 will actually be bounds on Λ. In our parameter analysis, large corrections to the relation in Eq. (32) would therefore only result in different labels along the f_a axes in our plots; the functional dependences displayed in these plots would still remain the same.

2.4   Stabilization of the axino and saxion

Next to the axion a itself, the chiral axion multiplet A in Eq. (29) also contains the superpartners of the axion: a two-component Weyl fermion, the axino $\tilde{a}$, as well as a real scalar, the saxion ϕ. These two particles are potentially produced in large numbers in the early universe by means of various thermal and nonthermal processes. This might have significant cosmological implications [59, 60]. Axino and saxion decays may, for instance, lead to the overproduction of dark matter, inject too much entropy into the thermal bath, thereby diluting the primordial baryon asymmetry, or alter the predictions of big bang nucleosynthesis. In order to prevent these catastrophic effects from taking place, either both the axino and saxion abundances need to be adequately suppressed or both species have to decay sufficiently fast. Here, the latter solution is, in particular, realized once $\tilde{a}$ and ϕ are given sufficiently large masses. On the other hand, one can show on rather general grounds that, in the supersymmetric limit, the entire axion multiplet A is necessarily massless [61]. In the case of unbroken SUSY, the saxion ϕ especially represents a flat direction in the scalar potential of the PQ-breaking sector, so that it is prone to cause cosmological problems. A successful stabilization of the PQ-breaking fields ($M_\pm$ in our case) can therefore only be achieved as long as $\tilde{a}$ and ϕ acquire appropriate soft masses in the course of spontaneous SUSY breaking. In the following, we shall show that this is exactly what is happening in our model.

First of all, let us trade the three SUSY-breaking singlet fields Z_+, Z_- and X in Eq. (14) for the following linear combinations,

$$S_0 = \frac{1}{(2-\zeta)^{1/2}} \left[(1-\zeta)^{1/2} (Z_+ + Z_-) - \zeta^{1/2} X \right] , \quad S_1 = \frac{1}{2^{1/2}} (Z_+ - Z_-) , \tag{34}$$

$$S_2 = \frac{1}{(2-\zeta)^{1/2}} \left[(\zeta/2)^{1/2} (Z_+ + Z_-) + 2^{1/2} (1-\zeta)^{1/2} X \right] .$$

As we will see shortly, these fields will end up corresponding to the physical mass eigenstates in the singlet sector. In terms of these new fields as well as in terms of the fields M and Θ in Eq. (25), the effective superpotential of the IYIT model in Eq. (14) can now be rewritten as follows,

$$W_{\text{eff}} \simeq \left[\mu^2 r^2 + \frac{m^2}{2\,\mu^2} \left(2 \, \text{ch}_\Theta \, F_A^2 - 2 \left(1 - \text{ch}_\Theta \right) F_A M - M^2 \right) \right] S_0 \tag{35}$$

$$- m \, \text{sh}_\Theta \, (F_A + M) \, S_1 - m \left[r \left(1 - \text{ch}_\Theta \right) (F_A + M) - \left(\frac{1}{r} + \frac{m^2}{2\,r\,\mu^4} F_A M \right) M \right] S_2 ,$$

13

where $\text{ch}_\Theta \equiv \cosh\Theta$ and $\text{sh}_\Theta \equiv \sinh\Theta$, where we have introduced $F_A \equiv K_0^{1/2} = 2^{-1/2}\,|\mathcal{A}_{\mathrm{PQ}}|\,f_a$ as an alternative symbol for the normalization of the axion multiplet (see Eq. (29)), where $\mu \equiv \Lambda_{\mathrm{SUSY}}$ denotes the SUSY breaking scale (see Eq. (22)) and where the parameters m and r are defined as

$$ m = \lambda_h \frac{\Lambda}{\eta}\,, \quad r = \left(\frac{\zeta}{2-\zeta}\right)^{1/2}\,. \tag{36}$$

From the superpotential in Eq. (35), we can calculate the scalar potential for the two scalar DOFs contained in $A = F_A\,\Theta \supset 2^{-1/2}\,(\phi + ia)$, i.e., for the axion a as well as for the saxion ϕ. As discussed in more detail in Appendix A, the VEVs of the three singlet fields S_0, S_1 and S_2 vanish in the rigid SUSY limit. Therefore, neglecting any SUGRA effects and setting all singlets to zero, the axion scalar potential induced by the spontaneous breaking of SUSY takes the following form,

$$ V(\phi,a) = \mu^4 r^2 + m^2 F_A^2 \cosh\left(\sqrt{2}\,F_A^{-1}\phi\right) = \mu^4 + \frac{1}{2}m_\phi^2\,\phi^2\left[1 + \frac{1}{6}\,(\phi/F_A)^2 + \mathcal{O}\left((\phi/F_A)^4\right)\right]\,. \tag{37}$$

Here, we have introduced $m_\phi^2 = 2\,m^2$ to denote the saxion mass and used the fact that the four parameters μ, m, F_A and r are actually not linearly independent; as one may easily check, they satisfy the relation $\mu^4 r^2 + m^2 F_A^2 = \mu^4$ (see Eqs. (22), (29) and (36)). The lesson from this scalar potential now is twofold. First of all, we note that the scalar potential $V(\phi,a)$ does indeed not depend on a, rendering the axion a flat direction. This is, of course, expected, given that the field a ought to represent the Nambu-Goldstone boson associated with the spontaneous breaking of the $U(1)_A$ symmetry by construction. Second, we find that the saxion indeed ends up being stabilized thanks to the SUSY-breaking dynamics of our model, $\langle\phi\rangle = 0$. Its mass around the origin is controlled by the mass parameter $m \propto \lambda_h\,\Lambda$, which is closely related to the SUSY breaking scale and which, moreover, goes to zero as soon as the SUSY-breaking sector decouples from the dynamics of PQ symmetry breaking (i.e., as soon as $\lambda_\pm \to 0$ for fixed $\kappa\,\eta$ in Eq. (14)).

In order to study the interactions of the axion multiplet A in the true vacuum, it is therefore sufficient to restrict our analysis to the superpotential in Eq. (35) in the limit of small fluctuations of the goldstone phase Θ around zero,

$$ W_{\mathrm{eff}} \simeq \mu^2 S_0 - m\,A\,S_1 + \frac{m}{r}\,M\,S_2 + \frac{m^2}{2\,\mu^2}\left(A^2 - M^2\right)S_0 \tag{38}$$

$$ - \frac{m}{2\,F_A}\left[2\,M\,A\,S_1 - \frac{1}{r}\,M^2 S_2 - r\left(A^2 - M^2\right)S_2\right] + \cdots\,, $$

where we have replaced Θ by A/F_A after expanding in powers of Θ and where the ellipsis stands for operators of dimension 4 and higher. This form of the superpotential provides us with a number of useful physical insights: (i) The singlet fields S_0, S_1 and S_2 indeed parametrize the fluctuations of the physical mass eigenstates around the true vacuum. Here, S_1 turns out to share a Dirac mass $m_A \equiv m$ with the axion field A, while S_2 turns out to share a Dirac mass $m_M \equiv m/r$ with the "radial" meson field M. Meanwhile, S_0 remains massless at tree level. (ii) As is now evident, the mass parameter m corresponds to the common Dirac mass of A and S_1, while r parametrizes the gap in the mass spectrum, i.e., the ratio between the two Dirac masses, $r = m_A/m_M$. (iii) Among

the three singlet fields, S_0 is the only one with a nonvanishing F-term. We can, thus, identify it with the goldstino (or Polonyi) multiplet which is responsible for the spontaneous breaking of SUSY via its F-term, $|F_{S_0}| = \mu^2$. Upon spontaneous SUSY breaking, its fermionic component, the goldstino $\tilde{s}_0$, is therefore absorbed by the gravitino $\tilde{G}$ (playing the role of its longitudinal DOFs thereafter), which is why it eventually acquires a mass $m_{\tilde{s}_0} \equiv m_{3/2}$. At the same time, the scalar component of the goldstino multiplet, the sgoldstino s_0, is a flat direction of the scalar potential at tree level, as it is present in any SUSY-breaking model of the O'Raifeartaigh type.

At the loop level, the (pseudo)modulus $s_0 \subset S_0$ is lifted via radiative corrections. The relevant loop diagrams arise from the $A^2 S_0$ and $M^2 S_0$ Yukawa interactions in Eq. (38) as well as from the Yukawa interactions of the X component of the goldstino field, $S_0 = -r\,X + \cdots$, with the neutral mesons in the full effective superpotential (i.e., from the $M_0^a M_0^b X$ terms in Eq. (12)). This results in the following contribution to the sgoldstino mass [62] (see Appendix A for details),[7]

$$m_0^2 = \frac{2\ln 2 - 1}{16\pi^2} \left[1 + \omega(r) + \frac{2}{\rho^6} \left(\left(\frac{\lambda_{14}}{\lambda} \right)^2 \omega_0\big(\lambda_0^{1,4}\big) + \left(\frac{\lambda_{23}}{\lambda} \right)^2 \omega_0\big(\lambda_0^{2,3}\big) \right) \right] \left(\frac{m}{\mu} \right)^4 m^2 , \quad (39)$$

$$\omega(r) = \frac{1}{2\ln 2 - 1} \left[\frac{1}{2}\left(1 + \frac{1}{r^2}\right)^2 \ln\left(1 + r^2\right) - \frac{1}{2}\left(1 - \frac{1}{r^2}\right)^2 \ln\left(1 - r^2\right) - \frac{1}{r^2} \right] \approx r^2 .$$

Here, the function ω, which smoothly interpolates between $\omega(0) = 0$ and $\omega(1) = 1$, acts as weight for the relative importance of the $M^2 S_0$ interaction compared to the $A^2 S_0$ interaction. In the case of a degenerate mass spectrum (i.e., for $r = 1$), diagrams with virtual M lines in the loop yield the same contribution to the sgoldstino mass as diagrams with virtual A lines. On the other hand, once the meson field M becomes much heavier than the axion field A (i.e., for $r \to 0$), the sgoldstino mass ceases to receive contributions from M loops. In this limit, saxion and axino loops then remain as the only source of mass generation for the sgoldstino in the charged meson sector. At the same time, the contributions to m_0^2 due to the interaction of S_0 with the neutral mesons are weighted by ω_0, evaluated as a function of the Yukawa couplings $\lambda_0^{1,4}$ and $\lambda_0^{2,3}$, respectively. The full expression for ω_0 is given in Appendix A. For now, we merely remark that, as long as $\lambda_0^a \geq \lambda$ for all a, also this weight smoothly interpolates between 0 and 1. Here, the maximal value, $\omega_0 = 1$, is, in particular, attained in the flavor-symmetric limit, i.e., for all λ_0^a being equal to λ. Furthermore, we note that the contributions to m_0^2 induced by the neutral meson loops come with prefactors proportional to λ_{14}^2 and λ_{23}^2, which are defined as follows,

$$\lambda_{14} = \left[\lambda^4 + \frac{1}{4}\left(\big(\lambda_0^1\big)^2 - \big(\lambda_0^4\big)^2 \right)^2 \right]^{1/4} , \quad \lambda_{23} = \left[\lambda^4 + \frac{1}{4}\left(\big(\lambda_0^2\big)^2 - \big(\lambda_0^3\big)^2 \right)^2 \right]^{1/4} . \quad (40)$$

---

[7]In [58], a similar expression for the loop-induced mass of the pseudoflat direction has been derived (see Eq. (57) in this paper). This expression does, however, not feature the weight function ω, as the authors of [58] only work in the limit $\kappa \to \infty$, where $\omega \to 0$. In this limit, the deformed moduli constraint is fulfilled exactly, the meson field M decouples completely and the $M^2 S_0$ interaction no longer contributes to m_0^2. Similarly, the $M_0^a M_0^b X$ interactions have been neglected in [58], so that the terms weighted by ω_0 are missing in this paper. Meanwhile, the calculation in [62] does account for the $M_0^a M_0^b X$ interactions. But, as it is also based on the assumption of an exactly fulfilled moduli constraint (i.e., on $\kappa \to \infty$), it, too, misses the contribution coming from the $M^2 S_0$ coupling.

In the limit of equal Yukawa couplings, $\lambda_0^a = \lambda$, the prefactors $(\lambda_{14}/\lambda)^2$ and $(\lambda_{23}/\lambda)^2$ therefore also reduce to unity, so that, in this limit, the sgoldstino mass in Eq. (39) takes the following form,

$$\lambda_0^a \equiv \lambda \quad \Rightarrow \quad m_0^2 = \frac{2\ln 2 - 1}{16\pi^2}\left[1 + \omega(r) + \frac{4}{\rho^6}\right]\left(\frac{m}{\mu}\right)^4 m^2, \quad \omega(r) \approx r^2. \tag{41}$$

For simplicity and since we do not expect any large flavor hierarchy in the IYIT model, we will work with this expression for m_0^2 (including the approximation $\omega(r) \approx r^2$) in the following.

In addition, working with an even more precise expression for m_0^2 (such as, for instance, the one in Eq. (39)) would not be of much help for another reason: Unfortunately, next to the perturbative Yukawa interactions encoded in the effective superpotential, the sgoldstino mass also receives contributions from the effective Kähler potential [62]. The true sgoldstino mass squared, $m_{s_0}^2$, is then given as the sum of m_0^2 and some dynamically generated and *uncalculable* correction,

$$m_{s_0}^2 = m_0^2 + \Delta m_{K_{\text{eff}}}^2. \tag{42}$$

For large Yukawa couplings, $\lambda \sim \eta$, we expect the uncalculable correction $\Delta m_{K_{\text{eff}}}^2$ to be of similar (but not much greater) importance as the perturbative result m_0^2. We note that this will be the more relevant case in the context of our phenomenological study later on (see Sec. 3). For smaller Yukawa couplings, $\lambda \ll \eta$, on the other hand, we have more confidence in the purely perturbative calculation. The upshot of these considerations is that the true sgoldstino mass is, most likely, always roughly of the order of the expression in Eq. (41), $m_{s_0} \sim m_0$. On top of that, if we further assume $\Delta m_{K_{\text{eff}}}^2$ to be positive, m_0 represents a lower bound on the actual sgoldstino mass,

$$\Delta m_{K_{\text{eff}}}^2 > 0 \quad \Rightarrow \quad m_{s_0}^2 \geq m_0^2. \tag{43}$$

This comes in handy, because it allows us to determine a *conservative* upper bound on the sgoldstino VEV after taking into account the effect of R symmetry breaking (see Appendix A). Such a conservative upper bound on $\langle S_0 \rangle$ is useful, since it prevents us from underestimating the impact of higher dimensional operators on the quality of the PQ symmetry (see Sec. 3).

(iv) Finally, a few comments on the masses of the remaining bosonic and fermionic DOFs contained in S_0, S_1, S_2, M, and A are in order. In the true vacuum and neglecting the effect of SUGRA on the VEV of the sgoldstino field S_0, the scalar masses in our model are given as follows,

$$m_{s_0^\pm}^2 \sim m_0^2, \quad m_\phi^2 = 2\,m^2, \quad m_a^2 = 0, \quad m_{s_1^\pm}^2 = m^2, \quad m_{m^\pm}^2 = \frac{m^2}{r^2}\left(1 \pm r^2\right), \quad m_{s_2^\pm}^2 = \frac{m^2}{r^2}. \tag{44}$$

Similarly, we obtain for the fermionic masses in the globally supersymmetric limit

$$m_{\tilde{s}_0}^2 = 0, \quad m_{\tilde{a}}^2 = m_{\tilde{s}_1}^2 = m^2, \quad m_{\tilde{m}}^2 = m_{\tilde{s}_2}^2 = \frac{m^2}{r^2}. \tag{45}$$

The dependence of these different mass eigenvalues on the Yukawa couplings λ and κ becomes more transparent, if we rewrite them as functions of $\zeta = \lambda^2/\left(\kappa^2\,\eta^2\right)$ (see Eqs. (15) and (36)),

$$\frac{m^2}{\rho^2\kappa^2\Lambda^2} = \zeta, \quad \frac{m^2/r^2}{\rho^2\kappa^2\Lambda^2} = 2 - \zeta, \quad \frac{m^2/r^2\left(1 + r^2\right)}{\rho^2\kappa^2\Lambda^2} = 2, \quad \frac{m^2/r^2\left(1 - r^2\right)}{\rho^2\kappa^2\Lambda^2} = 2\left(1 - \zeta\right), \tag{46}$$

These expressions allow us to study the SUSY-breaking and PQ-preserving limit ($\lambda \to \kappa \eta$) as well as the SUSY-preserving and PQ-breaking limit ($\lambda \to 0$) of our model in a nice fashion.[8] For $\lambda \to \kappa \eta$, the above masses squared (in units of $\rho^2 \kappa^2 \Lambda^2$) approach $\{1, 1, 2, 0\}$, while for $\lambda \to 0$, they turn into $\{0, 2, 2, 2\}$. Here, the massless field in the PQ-preserving limit (the real meson scalar m_-) is the result of an accidental cancellation in the scalar mass matrix for the special parameter choice $\lambda = \kappa \eta$. In the limit $\lambda \to \kappa \eta$, the field m_-, thus, becomes the second lightest state in the IYIT model, the only lighter field being the massless axion a. At the same time, the massless fields in the SUSY-preserving limit correspond to the DOFs contained in S_0, S_1, and A. We, hence, see once again that it is mandatory to break SUSY in order to stabilize the axino as well as the saxion.

Next, we note that, according to the above results for the mass eigenvalues in our model, the physical fields at low energies appear to correspond to four real scalars (ϕ, a, m_+, and m_-), three complex scalars (s_0, s_1, and s_2), one Weyl fermion ($\tilde{s}_0$) as well as two Dirac fermions (($\tilde{a}, \tilde{s}_1$) and ($\tilde{m}, \tilde{s}_2$)). In fact, all mass degeneracies in Eqs. (44) and (45) are, however, lifted through SUGRA effects—see Appendix A, where we derive the VEVs of all singlet fields taking into account the effect of R symmetry breaking and state the full expressions for all bosonic and fermion masses given a nonzero value of $\langle S_0 \rangle$. The fields in Eqs. (44) and (45) are therefore only quasi-degenerate, i.e., they are only degenerate in the rigid SUSY limit. In the full SUGRA case, we have to deal instead with ten real scalars, one Weyl fermion and four Majorana fermions. The mass splittings among the quasi-complex scalars and quasi-Dirac fermions is then of $\mathcal{O}\left(m_{3/2}\right)$ and therefore quite large. Last but not least, we mention that, imposing the deformed moduli constraint exactly, i.e., in the limit $\kappa \to \infty$, the fields contained in M and S_2 become formally infinitely heavy. In this limit, they are, thus, unphysical and need to be integrated out (see also the discussion in [58]). The effective superpotential of the IYIT model in Eq. (38) then turns into

$$W_{\text{eff}} \simeq \mu^2 S_0 - m \, A \, S_1 + \frac{m^2}{2 \, \mu^2} A^2 S_0 \,. \tag{47}$$

which is nothing but the superpotential studied in [58] (see Eq. (43) therein).

3   Quality of the PQ symmetry and phenomenological constraints

3.1   Protecting the PQ symmetry by means of an anomaly-free Z_4^R symmetry

Up to now, we have only discussed the renormalizable interactions among the fields of the IYIT model. In the context of SUGRA, we, however, expect gravitational effects at the Planck scale to induce further, nonrenormalizable interactions among these fields in the low-energy effective theory. *A priori*, there is no reason why these additional interactions should happen to respect the global PQ symmetry that is enjoyed by the IYIT model in the rigid SUSY limit. Instead,

---

[8]As for the sgoldstino, we have $m_0^2/(\rho^2 \kappa^2 \Lambda^2) = (2 \ln 2 - 1)(4 - 2\zeta + \rho^6)/\rho^2 \lambda^2/(8\pi^2) \, \zeta \, (2 - \zeta)^{-2}$, which turns into $(2 \ln 2 - 1)(2 + \rho^6)/\rho^2 \lambda^2/(8\pi^2)$ for $\lambda \to \kappa \eta$ ($\simeq 2.3$ for $\lambda = 4\pi$ and $\rho = 1$) and into 0 for $\lambda \to 0$. We also recall that $m_{3/2}^2/(\rho^2 \kappa^2 \Lambda^2) = (\Lambda^2/\eta^2)/(3M_{\text{Pl}}^2)/\rho^2 \zeta \, (2 - \zeta)$. This goes to $(\Lambda^2/\eta^2)/(3M_{\text{Pl}}^2)/\rho^2$ for $\lambda \to \kappa \eta$ and to 0 for $\lambda \to 0$.

the full effective superpotential as well as the full effective Kähler potential at energies below the dynamical scale are expected to contain higher-dimensional operators that explicitly break PQ,

$$W_{\text{eff}}^{\cancel{PQ}} \supset \frac{\Lambda^2}{M_*} M_\pm^2 , \ \frac{\Lambda^3}{M_*^3} M_\pm^3 , \ \frac{1}{M_*} Z_\pm^4 , \ \cdots , \quad K_{\text{eff}}^{\cancel{PQ}} \supset \frac{\Lambda^2}{M_*^2} M_\pm^2 , \ \frac{\Lambda^3}{M_*^4} M_\pm^3 , \ \frac{1}{M_*} Z_\pm^3 , \ \cdots , \quad (48)$$

with M_* denoting an appropriate high-energy cut-off scale close to the Planck scale, $M_* \sim M_{\text{Pl}}$. These operators result in corrections to the ordinary axion potential in QCD, which causes the axion VEV to shift from its desired value, $\langle a \rangle = f_a \bar{\theta}$, to some displaced value, $\langle a \rangle = f_a (\bar{\theta} + \Delta\bar{\theta})$, at which CP is no longer conserved. In other words, the gravity-induced higher-dimensional operators in the effective theory re-introduce a nonzero QCD vacuum angle, $\Delta\bar{\theta}$, (which may easily become very large, $\Delta\bar{\theta} \gg 10^{-10}$) and, hence, bring us back to the original strong CP problem.

In order to suppress $\Delta\bar{\theta}$ below the experimental bound, $\Delta\bar{\theta} \lesssim 10^{-10}$, it is necessary to forbid all effective operators that explicitly violate PQ up to some high order. As discussed in the introduction (see also [34]), this is best done by invoking a protective gauge symmetry that eliminates all of the relevant dangerous operators from the effective theory. In this paper, we shall, in particular, rely on a discrete Z_4^R symmetry, which is well motivated from the perspective of PGM (see our discussion in Sec. 1.2). Let us now derive the charge spectrum for such a Z_4^R symmetry in the context of the IYIT model and assess which PQ-breaking operators in the effective theory it is able for forbid. The characteristics of general Z_N^R symmetries along with the charge assignment for the MSSM fields have already been reviewed in [34], which is why we will be rather brief in what follows. First of all, let us group the fields of the MSSM into complete multiplets of $SU(5)$, $\mathbf{10} = (q, u^c, e^c)$, $\mathbf{5^*} = (d^c, \ell)$, and $\mathbf{1} = (n^c)$, since we are only interested in charge assignments that are at least compatible with $SU(5)$ unification. The MSSM fields are then charged under the discrete Z_4^R symmetry as follows (for details, see Sec. 2.2.2 and Appendix A in [34]),

$$(r_{\mathbf{10}}, r_{\mathbf{5^*}}, r_{\mathbf{1}}, r_{H_u}, r_{H_d}) \overset{(4)}{=} \frac{1}{5} (1, -3, 5, 8, 12) + \frac{2\alpha}{5} (1, -3, 5, -2, 2) . \quad (49)$$

Here, α is an integer that can take any value between 0 and 9. Also, notice that the row vector multiplied by $2\alpha/5$ on the right-hand side of Eq. (49) encompasses the charges of the MSSM fields under the Abelian GUT group $U(1)_X$. This group, sometimes referred to as "fiveness", commutes with $SU(5)$ and can be represented as a linear combination of the weak hypercharge Y and the difference between baryon number B and lepton number L, i.e., $X = 5(B-L) - 4Y$. We note that the MSSM R charges are not uniquely defined, since the MSSM superpotential (including Majorana mass terms for the singlet neutrino fields n^c) happens to be invariant under $Z_{10} \subset U(1)_X$ transformations. This ambiguity leaves us with ten different solutions for the MSSM R charges.

Next, we point out that, solely within the MSSM, the Z_4^R symmetry turns out to be anomalously violated at the quantum level for every possible R charge assignment. This is illustrated by the fact that the color as well as the weak anomaly coefficient for the Z_4^R symmetry are always nonzero,

$$\mathcal{A}_R^{(C)} = \mathcal{A} \left[Z_4^R\text{--}SU(3)_C\text{--}SU(3)_C \right] = 6 + N_g \left(3\, r_{\mathbf{10}} + r_{\mathbf{5^*}} - 4 \right) , \quad (50)$$

$$\mathcal{A}_R^{(L)} = \mathcal{A} \left[Z_4^R\text{--}SU(2)_L\text{--}SU(2)_L \right] = 4 + N_g \left(3\, r_{\mathbf{10}} + r_{\mathbf{5^*}} - 4 \right) + \left(r_{H_u} + r_{H_d} - 2 \right) ,$$

where $N_g = 3$ stands for the number of SM fermion generations. To see that $\mathcal{A}_R^{(C)}$ and $\mathcal{A}_R^{(L)}$ are indeed nonzero (without inserting all possible R charges into Eq. (50) by hand), we have to employ Eq. (3) as well as the following relations between the charges r_{10}, r_{5^*}, r_{H_u}, and r_{H_d},

$$2\, r_{10} + r_{H_u} \stackrel{(4)}{=} 2 \,, \quad r_{5^*} + r_{10} + r_{H_d} \stackrel{(4)}{=} 2 \,, \tag{51}$$

which directly result from the MSSM Yukawa interactions. We then find for $\mathcal{A}_R^{(C)}$ and $\mathcal{A}_R^{(L)}$,

$$\mathcal{A}_R^{(C)} \stackrel{(4)}{=} \mathcal{A}_R^{(L)} \stackrel{(4)}{=} 6 - 4\, N_g \stackrel{(4)}{=} -2 \,. \tag{52}$$

In order to cancel the MSSM contributions to $\mathcal{A}_R^{(C)}$ and $\mathcal{A}_R^{(L)}$ and, hence, render the Z_4^R symmetry anomaly-free, we are therefore led to introduce new matter multiplets that transform under $SU(5)$. Here, the easiest possibility is to simply add a certain number of $\mathbf{5}$ and $\mathbf{5}^*$ representations, which we shall refer to as Q_i and $\bar{Q}_i$ in the following. Given $k \in \mathbb{N}$ of such new "quark/antiquark" pairs, the total R charge of the extra matter fields needs to satisfy

$$r_{Q\bar{Q}} = r_Q + r_{\bar{Q}} \stackrel{(4)}{=} 2 - \Delta r \,, \quad \Delta r = -\frac{1}{k}\left(2 + 4\ell\right) \,, \quad \ell = 0, 1, \cdots k - 1 \,, \tag{53}$$

so as to make the Z_4^R anomaly coefficients vanish. We, thus, conclude that our axion model predicts the existence of new SM-charged fields, without the aid of which we would not able not invoke an anomaly-free discrete Z_4^R symmetry as a protective gauge symmetry for the PQ symmetry.

Moreover, the new matter fields Q_i and $\bar{Q}_i$ come in handy for another reason. In order to allow for a successful solution of the strong CP problem, the PQ symmetry in the IYIT sector needs to exhibit a color anomaly. This is now easily achieved by coupling Q_i and $\bar{Q}_i$ to the SUSY-breaking sector. For instance, and w.l.o.g., we may assume that all $Q\bar{Q}$ pairs couple to $\Psi^1 \Psi^2$ via some Planck-suppressed operators. Above and below the dynamical scale, we then respectively have

$$W^Q = \sum_{i=1}^{k} \frac{C'_{Q_i}}{M_*} \left(Q\bar{Q}\right)_i \Psi^1 \Psi^2 \,, \quad W_{\text{eff}}^Q \simeq \sum_{i=1}^{k} \frac{C_{Q_i}}{M_*} \frac{\Lambda}{\eta} \left(Q\bar{Q}\right)_i M_+ \,, \tag{54}$$

where the dimensionless coefficients C'_{Q_i} and C_{Q_i} are naively expected to be of $\mathcal{O}(1)$ or at most as large as 4π. In view of the superpotential terms in Eq. (54), three comments are now in order: (i) Since the meson field M_+ carries PQ charge $+1$, each quark/antiquark pair must carry PQ charge -1. Meanwhile, as no MSSM field couples to the SUSY-breaking sector directly, all MSSM fields remain uncharged under the PQ symmetry. The total PQ color anomaly, hence, receives contributions from the new quark fields only,

$$\mathcal{A}_{\text{PQ}}^{(C)} = k\left(q_Q + q_{\bar{Q}}\right) = -k \,, \quad q_Q + q_{\bar{Q}} = -1 \,. \tag{55}$$

This renders our axion model a special supersymmetric variant of the KSVZ axion model invented by Kim, Shifman, Vainshtein, and Zakharov a long time ago [9]. (ii) The superpotential couplings in Eq. (54) also act as mass terms for the new quark fields. In fact, upon spontaneous PQ symmetry breaking, each quark pair acquires a supersymmetric Dirac mass close to the gravitino mass,

$$m_{Q_i} = \frac{C_{Q_i}}{M_*} \frac{\Lambda}{\eta} \langle M_+ \rangle = \frac{C_{Q_i}}{\lambda_+} \frac{\sqrt{3}\, M_{\text{Pl}}}{M_*} \left(\frac{1 - \zeta}{2 - \zeta}\right)^{1/2} m_{3/2} \,. \tag{56}$$

19

Therefore, depending on the value of $m_{3/2}$, the extra quark fields may or may not be light enough to be within the reach of a future multi-TeV collider experiment. Albeit extremely challenging, the discovery of $\mathcal{O}(k)$ new $SU(5)$ multiplets in the vicinity of the gravitino mass would then, of course, be a smoking-gun signal of our axion model.

(iii) Last but not least, we note that the form of the superpotential in Eq. (54) together with the IYIT superpotential in Eq. (12) suffices to fix the R charges of all fields in the IYIT sector,

$$r_{M_\pm} = \pm\Delta r \,, \quad r_{Z_\pm} = 2 \pm \Delta r \,, \quad r_{M_0^a} = 0 \,, \quad r_{Z_0^a} = r_X = 2 \,. \tag{57}$$

Here, we have required that the Z_4^R symmetry be anomaly-free under the strongly coupled $SU(2)$. Simply by itself, this implies that the charges of all IYIT quarks must sum to zero: $r_{M_+} + r_{M_-} = 0$ and, thus, $r_X = 2$. Notice that, if this was not the case, the dynamically generated superpotential (see Eq. (6)) would explicitly break R symmetry, $Z_4^R \to \emptyset$, so that we could no longer rely on R symmetry as a tool to constrain the low-energy effective theory. In particular, we would loose control over the dynamically generated terms in the superpotential and Kähler potential, which might lead to too large a gravitino mass or other unwanted effects. Moreover, we point out that our result in Eq. (57) reveals an interesting relation between the charges of the fields in the IYIT sector under the local Z_4^R symmetry and the charges of the same fields under the continuous global R symmetry of the IYIT model in the rigid SUSY limit. Under the latter, all meson fields are uncharged, while all singlet fields carry charge $+2$. If we denote these global R charges by r^0 for the individual fields (and if we denote the corresponding PQ charges by q), we arrive at

$$r = r^0 + q\,\Delta r \,, \tag{58}$$

which holds for every field in the SUSY-breaking sector. This is to say that, invoking an anomaly-free Z_4^R symmetry in the IYIT sector, we are actually doing nothing else but gauging a discrete subgroup of the global $U(1)_R \times U(1)_A$ symmetry of the IYIT model. As we shall demonstrate in the next section, this discrete gauge symmetry then allows us to eliminate dangerous higher-dimensional operators in the effective theory. At the same time, it also fixes the structure of the renormalizable interactions in the IYIT model. Without invoking any further symmetry, the renormalizable superpotential and Kähler potential could also contain terms such as

$$W_{\mathrm{eff}}^{\mathrm{PQ}} \supset Z_\pm^2 \,, \ Z_\pm^3 \,, \ \cdots \,, \quad K_{\mathrm{eff}}^{\mathrm{PQ}} \supset Z_\pm \,, \ Z_\pm^2 \,, \ \cdots \,. \tag{59}$$

So far, we have simply ignored this issue; and now we see that, in general (i.e., for most values of the two integers k and ℓ), such terms are automatically forbidden by the Z_4^R symmetry.

Finally, we mention that, as a result of the relation in Eq. (58), the SUSY-breaking sector on its own turns out not to break the Z_4^R symmetry—even though the charged meson fields $M_\pm$ carry nonzero R charge and obtain large VEVs. Here, the point is that we can always rotate away the charges of the charged meson fields by means of a global PQ phase transformation, such that $r \to r' \equiv r^0$. The strong dynamics of the IYIT sector therefore only break SUSY as well as the global PQ symmetry, but leave the gauged Z_4^R intact, though, $Z_4^R \times U(1)_{\mathrm{PQ}} \to Z_4^{R\prime}$. This remnant

$Z_4^{R\prime}$ symmetry is then only broken, $Z_4^{R\prime} \to Z_2^R$, by the constant term in the superpotential,[9] $W_0 = m_{3/2}\,M_{\mathrm{Pl}}^2$, as well as by higher-dimensional operators in the effective theory (such as the quark mass term in Eq. (54)). Here, it is interesting to observe that the Z_2^R parity that we are eventually left with can be identified with the R parity of the MSSM. Our model therefore automatically accounts for the origin of R parity in the MSSM. That is, in contrast to many other models, it does not require any extension by, say, a gauged $B-L$ symmetry to do so [64].

3.2 Constraints on the axion decay constant

We have not yet uniquely specified all properties of the Z_4^R symmetry. Our construction still exhibits two free parameters: k, the number of extra quark pairs, as well as Δr, the shift in the global R charges r^0 (see Eqs. (53) and (58)). In the following, we shall now examine for which values of these parameters we have a chance of arriving at a viable phenomenology as well as how the other parameters of our model (the axion decay constant f_a, the gravitino mass $m_{3/2}$, the Yukawa coupling λ, etc.) are respectively constrained in these different scenarios.

First of all, we note that, in order to forbid as many PQ-breaking operators as possible, it turns out advantageous to choose the integer ℓ in Eq. (53) such that Δr ends up being a fraction and not an integer, $\Delta r \notin \mathbb{N}$. This already rules out scenarios with only one or two extra quark pairs from the start[10] and implies that only the k-th powers of the fields $M_\pm$ and $Z_\pm$ can appear in the effective superpotential as well as the effective Kähler potential. The crucial point here is that, for $\Delta r \notin \mathbb{N}$, the smallest integer multiple of Δr is nothing but k times Δr,

$$k\,\Delta r \stackrel{(4)}{=} 2\,. \tag{60}$$

For even and odd values of k, the lowest-dimensional PQ-breaking operators in $W_{\mathrm{eff}}^{\mathrm{PQ}}$ and $K_{\mathrm{eff}}^{\mathrm{PQ}}$ are then respectively given as follows,

$$W_{\mathrm{eff}}^{\cancel{\mathrm{PQ}}} \supset \begin{cases} M_\pm^k\,,\ \ Z_\pm^k & ;\ k\ \text{even} \\ M_\pm^k\,,\ \ m_{3/2}\,Z_\pm^k & ;\ k\ \text{odd} \end{cases}, \quad K_{\mathrm{eff}}^{\cancel{\mathrm{PQ}}} \supset \begin{cases} m_{3/2}\,M_\pm^k\,,\ \ m_{3/2}\,Z_\pm^k & ;\ k\ \text{even} \\ m_{3/2}\,M_\pm^k\,,\ \ Z_\pm^k & ;\ k\ \text{odd} \end{cases}. \tag{61}$$

Recall that the mesons $M_\pm$ acquire VEVs of $\mathcal{O}\left(\Lambda\right)$ (see Eq. (15)), while, in the context of SUGRA, the singlets Z_+ and Z_- obtain VEVs of $\mathcal{O}\left(m_{3/2}\right)$ (see Eq. (124)). Together with $m_{3/2} \sim \Lambda^2/M_{\mathrm{Pl}}$ (see Eq. (24)), these estimates allow us to assess the order of magnitude of the respectively most important corrections to the axion scalar potential, ΔV_a, induced by these PQ-breaking operators,

$$W_{\mathrm{eff}}^{\cancel{\mathrm{PQ}}} \ \to \ \Delta V_a \sim \left(\frac{m_{3/2}}{M_{\mathrm{Pl}}}\right)^{k+1-c} M_{\mathrm{Pl}}^4\,, \quad K_{\mathrm{eff}}^{\cancel{\mathrm{PQ}}} \ \to \ \Delta V_a \sim \left(\frac{m_{3/2}}{M_{\mathrm{Pl}}}\right)^{k+2+c} M_{\mathrm{Pl}}^4\,, \tag{62}$$

---

[9]If the constant term in the superpotential is generated at very high energies (for instance, via gaugino condensation [63]), we do not have to worry about any cosmological consequences of R symmetry breaking. In such a case, all dangerous topological defects created during R symmetry breaking will simply be inflated away.

[10]For $k = 1$, we are unable to forbid tadpole terms for the singlet fields $Z_\pm$ in the renormalizable Kähler potential, $K \supset Z_\pm$, while for $k = 2$, we are unable to forbid supersymmetric mass terms for the same fields in the renormalizable superpotential, $W \supset Z_\pm^2$. These scenarios are, therefore, unfeasible from the very beginning.

where $c = 1$ for even k and $c = 0$ for odd k. Here, notice that the meson operators require a different power counting than the singlet operators (see also Eq. (48)). As the meson fields are, in fact, composite fields, $M^{ij} \sim \Psi^i \Psi^j / \Lambda$ (see Eq. (4)), each meson field is actually accompanied by one power of the dynamical scale, so that each meson VEV is bound to come with a suppression factor of $\mathcal{O}(\Lambda / M_{\mathrm{Pl}})$. Therefore, despite the hierarchy between the actual VEVs, $\langle M_\pm \rangle \gg \langle Z_\pm \rangle$, the effect of the respective meson and singlet operators ends up being comparable,[11]

$$\frac{\Lambda}{M_{\mathrm{Pl}}} \langle M_\pm \rangle \sim \frac{\Lambda^2}{M_{\mathrm{Pl}}} \sim m_{3/2} \sim \langle Z_\pm \rangle \; . \tag{63}$$

The main lesson from Eq. (62) is that all PQ-breaking effects induced by the effective Kähler potential in Eq. (61) are suppressed compared to the corresponding effects induced by the effective superpotential by at least one power of the ratio $m_{3/2}/M_{\mathrm{Pl}}$. This is perhaps not much of a surprise, since the Kähler potential in Eq. (61) is holomorphic in the fields $M_\pm$ and $Z_\pm$, so that it can only contribute to the total scalar potential via pure SUGRA terms. By comparison, the lowest-dimensional *nonholomorphic* terms in $K_{\mathrm{eff}}^{\mathrm{PQ}}$ are obtained by multiplying the terms in Eq. (61) by the R-invariant field products $M_\pm M_\pm^*$ and $Z_\pm Z_\pm^*$, respectively. These higher-dimensional terms then yield corrections to the axion scalar potential which are of the same order of magnitude as the corrections induced by the holomorphic terms in $K_{\mathrm{eff}}^{\mathrm{PQ}}$. In summary, we therefore find that the PQ-breaking effects stemming from the Kähler potential are always suppressed and that it suffices to focus on the PQ-breaking operators contained in the superpotential in the following.

Let us now be a bit more specific and write down the operators in $W_{\mathrm{eff}}^{\mathrm{PQ}}$ in Eq. (48) including all prefactors, powers of the dynamical scale Λ, powers of the cut-off scale M_*, etc.,

$$W_{\mathrm{PQ}}^{\mathrm{eff}} \simeq \frac{C_{Z_\pm}}{k!} \left(1 \text{ or } \frac{m_{3/2}}{M_*} \right) \frac{Z_\pm^k}{M_*^{k-3}} + \frac{C_{M_\pm}}{(k!)^2} \frac{1}{\eta^2} \left(\frac{\eta \Lambda}{M_*} \right)^k \frac{M_\pm^k}{M_*^{k-3}} \; , \tag{64}$$

with the coefficients $C_{Z_\pm}$ and $C_{M_\pm}$ denoting some unknown constants of $\mathcal{O}(1)$ and where the prefactor of $Z_\pm^k$ is determined by whether the integer k is chosen to be even or odd (see Eq. (48)). The most dangerous corrections to the axion scalar potential resulting from this superpotential (deriving from F-term contributions well as from A-term contributions in SUGRA) are the following,

$$\Delta V_a = \frac{C_{Z_\pm}}{(k-1)!} \lambda_\pm \left(1 \text{ or } \frac{m_{3/2}}{M_*} \right) \frac{\Lambda}{\eta} \frac{M_\pm^* Z_\mp^{k-1}}{M_*^{k-3}} \tag{65}$$

$$+ \frac{C_{M_\pm}}{(k!)^2} \frac{1}{\eta^2} \left(\frac{\eta \Lambda}{M_*} \right)^k \left(k \, \kappa \, \eta \, X^* M_\mp^* + k \, \lambda_\pm \frac{\Lambda}{\eta} Z_\mp^* + (k-3) \, m_{3/2} \, M_\pm \right) \frac{M_\pm^{k-1}}{M_*^{k-3}} + \text{h.c.} \, ,$$

[11]This different power counting in the case of the meson operators represents a distinctive feature of our *dynamical* axion model, which distinguishes it from our earlier axion models presented in [34]. In this earlier work, the PQ-breaking fields are taken to be elementary fields, $M_\pm \rightarrow P, \bar{P}$, so that their VEVs do not end up being suppressed by a factor of $\mathcal{O}(\Lambda / M_{\mathrm{Pl}})$. This allows, *inter alia*, for the possibility of extra quark fields as heavy as the dynamical scale, $m_{Q_i} \sim \Lambda$, and increases the magnitude of the PQ-breaking terms in the axion potential. In the present paper, the mass scale of the new quark fields is, by contrast, tied to the gravitino mass, $m_{Q_i} \sim \Lambda^2 / M_{\mathrm{Pl}} \sim m_{3/2}$, and the PQ-breaking terms in the axion potential are generally more strongly suppressed. We emphasize that it is these differences that explain why we cannot simply use the results for general Z_N^R symmetries obtained in [34] and apply them to the present scenario in the special case of a Z_4^R symmetry. Instead, a new and dedicated study is necessary.

where all chiral fields are understood to represent their scalar components. In order to make the dependence of these terms on the axion field value a manifest, we need to expand the charged fields $M_\pm$ and $Z_\pm$ around their VEVs (see Eq. (25)). Taking into account the fact that $\langle Z_\pm \rangle \neq 0$ in SUGRA (see Appendix A), we then have for the complex scalars contained in $M_\pm$ and $Z_\pm$,

$$M_\pm = \langle M_\pm \rangle \exp\left(\pm \frac{i\,a}{\sqrt{2}\,F_A}\right), \quad Z_\pm = \langle Z_\pm \rangle \exp\left(\pm \frac{i\,a}{\sqrt{2}\,F_A}\right), \tag{66}$$

where we have set all further scalar DOFs contained in $M_\pm$ and $Z_\pm$ to zero. In passing, we also mention that, in SUGRA, the scale F_A also receives contributions from the singlet fields $Z_\pm$,

$$F_A = K_0^{1/2}, \quad K_0 = \left\langle |M_+|^2 \right\rangle + \left\langle |M_-|^2 \right\rangle + \left\langle |Z_+|^2 \right\rangle + \left\langle |Z_-|^2 \right\rangle. \tag{67}$$

However, since the VEVs of the singlets are much smaller than the meson VEVs, this is only a small correction compared to the globally supersymmetric case. In the following, we shall therefore neglect the SUGRA corrections to F_A in Eq. (67) and simply work with the expression in Eq. (29).

Plugging the expressions in Eq. (66) into the scalar potential in Eq. (65), we find that all dangerous operators in the axion scalar potential can be brought into the following form,

$$\Delta V_a \supset \frac{1}{2}\,v^4\left[\exp\left(\pm i\,\frac{k\,a}{\sqrt{2}\,F_A}\right) + \text{h.c.}\right] = v^4 \cos\left(\frac{k\,a}{\sqrt{2}\,F_A}\right), \quad v \sim \left(\frac{m_{3/2}}{M_{\rm Pl}}\right)^{(k+1-c)/4} M_{\rm Pl}, \tag{68}$$

for some appropriate mass scale v that differs from operator to operator. This correction to the scalar potential needs to be compared with the instanton-induced scalar axion potential in QCD,

$$V_a^{(0)} \simeq m_a^2\,f_a^2\left[1 - \cos\left(\bar\theta - \frac{a}{f_a}\right)\right], \quad m_a = \frac{z^{1/2}}{1+z}\frac{m_\pi f_\pi}{f_a} \simeq 600\,\mu\text{eV}\left(\frac{10^{10}\,\text{GeV}}{f_a}\right), \tag{69}$$

with m_a denoting the axion mass in QCD, which is determined by the π^0 mass $m_{\pi^0} \simeq 135\,\text{MeV}$, the π^0 decay constant $f_{\pi^0} \simeq 92\,\text{MeV}$, the ratio of the up and the down quark mass, $z = m_u/m_d \simeq 0.56$, as well as by the axion decay constant f_a [65]. The sum of $V_a^{(0)}$ and ΔV_a is then no longer minimized at the CP-conserving field value $\langle a \rangle = f_a\,\bar\theta$, but rather at $\langle a \rangle = f_a\left(\bar\theta + \Delta\bar\theta\right)$, where

$$\Delta\bar\theta = \Delta\bar\theta_0 \sin\left(\frac{k}{|\mathcal{A}_{\rm PQ}|}\bar\theta\right) = \Delta\bar\theta_0 \sin\bar\theta, \quad \Delta\bar\theta_0 = \frac{k}{|\mathcal{A}_{\rm PQ}|}\frac{v^4}{m_a^2\,f_a^2} = \frac{v^2}{m_a^2\,f_a^2}, \tag{70}$$

up to corrections of $\mathcal{O}\left(\Delta\bar\theta_0^2\right)$ and where we have used that $|\mathcal{A}_{\rm PQ}| = k$ (see Eq. (55)). According to the experimental bound on the QCD angle, $\Delta\bar\theta_0$ must be smaller than 10^{-10}, which leads us to

$$v \lesssim 10^{-2.5}\,\Lambda_a \simeq 240\,\text{keV}, \quad \Lambda_a = (m_a\,f_a)^{1/2} \simeq 77\,\text{MeV}. \tag{71}$$

The energy scale of the PQ-breaking operators in the axion potential therefore needs to be extremely suppressed, i.e., it should be even smaller than half the electron mass! Given our estimate of the energy scale v in Eq. (68) and taking the gravitino mass to be of $\mathcal{O}(100)\,\text{TeV}$, this implies that scenarios with only $k = 3$ or $k = 4$ pairs of extra quarks can be safely ruled out,

$$\left(\frac{m_{3/2}}{M_{\rm Pl}}\right)^{(k+1-c)/4} M_{\rm Pl} \lesssim 240\,\text{keV}, \quad m_{3/2} \sim 100\,\text{TeV}, \quad \Rightarrow \quad k \geq k_{\min} \sim 5 \tag{72}$$

Whether or not $k = 5$ extra quark pairs are phenomenologically viable is hard to tell in view of this rather simplified estimate. The case $k = 5$, thus, requires a more careful analysis. In fact, as we shall see in the following, it turns out that $k = 5$ new quark pairs are not only viable, but also the *unique* number of new quark pairs that will allow us to satisfy all bounds at the same time.

In order to constrain scenarios with $k \geq 5$ extra quark pairs more precisely, we need to know the exact expressions for the energy scale v belonging to the respective terms in ΔV_a in Eq. (65). These expressions simply follow from substituting all fields in Eq. (65) with their VEVs (see Eq. (15) as well as Eq. (124) in Appendix A for our results for $\langle M_\pm \rangle$, $\langle Z_\pm \rangle$, and $\langle |X| \rangle$, respectively),

$$v_{Z_\pm}^4 = \frac{2\, C_{Z_\pm}}{(k-1)!}\, \lambda_\pm \left(1 \text{ or } \frac{m_{3/2}}{M_*} \right) \frac{\Lambda}{\eta} \frac{\langle M_\pm \rangle \langle Z_\mp \rangle^{k-1}}{M_*^{k-3}}\,, \tag{73}$$

$$v_{M_\pm}^4 = \frac{2\, C_{M_\pm}}{(k!)^2} \frac{1}{\eta^2} \left(\frac{\eta\, \Lambda}{M_*} \right)^k \left(k\, \kappa\, \eta\, \langle |X| \rangle \langle M_\mp \rangle + k\, \lambda_\pm \frac{\Lambda}{\eta}\, \langle Z_\mp \rangle + (k-3)\, m_{3/2}\, \langle M_\pm \rangle \right) \frac{\langle M_\pm \rangle^{k-1}}{M_*^{k-3}}\,,$$

where the additional factors of 2 cancel with the factor $1/2$ in Eq. (68). Imposing the requirement that these scales be sufficiently suppressed compared to the "axion scale" (see Eqs. (70) and (71)),

$$v_{Z_+}^4 + v_{Z_-}^4 \lesssim \Delta\bar{\theta}_0^{\max}\, \Lambda_a^4\,, \quad v_{M_+}^4 + v_{M_-}^4 \lesssim \Delta\bar{\theta}_0^{\max}\, \Lambda_a^4\,, \quad \Delta\bar{\theta}_0^{\max} = 10^{-10}\,, \tag{74}$$

we are then able to derive two k-dependent upper bounds on the axion decay constant f_a,

$$Z_\pm^k \;\to\; f_a \lesssim f_Z^{(k)} = A_Z^{(k)}(\zeta, \rho, \kappa, \eta)\, F_Z^{(k)}\,, \quad F_Z^{(k)} = (M_{\mathrm{Pl}}\, M_*)^{1/2} \left(\frac{\Delta\bar{\theta}_0^{\max}\, \Lambda_a^4}{C_Z\, M_{\mathrm{Pl}}\, M_*^3} \right)^{1/(2(k+1-c))}\,, \tag{75}$$

$$M_\pm^k \;\to\; f_a \lesssim f_M^{(k)} = A_M^{(k)}(\zeta, \rho, \kappa, \eta)\, F_M^{(k)}\,, \quad F_M^{(k)} = M_* \left(\frac{\Delta\bar{\theta}_0^{\max}\, \Lambda_a^4\, M_{\mathrm{Pl}}}{C_M\, M_*^5} \right)^{1/(2(k+1))}\,,$$

Here, $A_Z^{(k)}$ and $A_M^{(k)}$ represent two dimensionless prefactors, the precise values of which depend on four crucial parameters of our model: ζ (i.e., the Yukawa coupling λ in the IYIT superpotential, see Eq. (15)), ρ (i.e., the flavor hierarchy in the IYIT sector, see Eq. (29)), κ (i.e., the physical status of the Lagrange multiplier field X, see Eq. (6)), and η (i.e., the numerical uncertainty of all coupling constants in the effective theory induced by strong-coupling effects, see Eq. (4)),

$$k \text{ even:} \quad A_Z^{(k)}(\zeta, \rho, \kappa, \eta) = \left[\frac{3^{(k-1)/2}\, k!\, (\kappa\eta)^{2k-3}}{(2\pi)^{2(k-1)}\, k^{2k+1}\, \rho^{2k}} \frac{\zeta^{k-3/2}\, (1-\zeta)^{k/2}}{(2-\zeta)^{5(k-1)/2}}\, B^{k-1} \right]^{1/(2k)}\,, \tag{76}$$

$$k \text{ odd:} \quad A_Z^{(k)}(\zeta, \rho, \kappa, \eta) = \left[\frac{4 \times 3^{k/2}\, k!\, (\kappa\eta)^{2(k-2)}}{(2\pi)^{2(k-1)}\, k^{2k+3}\, \rho^{2(k+1)}} \frac{\zeta^{k-2}\, (1-\zeta)^{k/2+1}}{(2-\zeta)^{5k/2-2}}\, B^{k-1} \right]^{1/(2k+2)}\,,$$

$$A_M^{(k)}(\zeta, \rho, \kappa, \eta) = \left[\frac{2^{2k+1}\, 3^{1/2}\, k!^2\, \kappa\eta\, C^{k/2}}{k^{2(k+1)}\, \eta^{2(k-1)}\, \rho^{k+2}} \frac{\zeta^{1/2}\, (1-\zeta)^{k/2+1}}{(2-\zeta)^{1/2}} \frac{B}{D} \right]^{1/(2k+2)}\,,$$

where we have introduced the symbols B, C, and D for the ease of notation,

$$B = (2\ln 2 - 1)\left(4 - 2\zeta + \rho^6 \right)\,, \tag{77}$$

$$C = 1 + \left(1 - \rho^4 \right)^{1/2}\,,$$

$$D = 32\pi^2\, k\, (2-\zeta)^2 \left[\rho^{2k-2}C + \rho^{-2}\,(2-C)\,C^k \right] + (k-3)\,(\kappa\eta)^2\, \zeta\, B \left(\rho^{2k} + C^k \right)\,.$$

It is illustrative to evaluate the two bounds in Eq. (75) for a few representative parameter values. For $k = 5$ and $k = 6$, for instance, and setting $C_Z = C_M = 1$ as well as $M_* = M_{\mathrm{Pl}}$, the two energy scales $F_Z^{(k)}$ and $F_Z^{(k)}$ in Eq. (75) take the following values,

$$F_Z^{(5)} = F_Z^{(6)} = F_M^{(5)} \simeq 1.1 \times 10^{11}\,\mathrm{GeV}\,, \quad F_M^{(6)} \simeq 1.3 \times 10^{12}\,\mathrm{GeV}\,, \tag{78}$$

which is well above the lower astrophysical bound on the axion decay constant, $f_a \gtrsim 10^9\,\mathrm{GeV}$ (see Sec. 1.1). At the same time, for $\kappa = 1$, $\eta = 4\pi$ and assuming identical Yukawa couplings in the IYIT superpotential (i.e., $\rho = 1$), $A_Z^{(5,6)}$ and $A_M^{(5,6)}$ are all of $\mathcal{O}(0.1)$ for almost all values of ζ,

$$A_Z^{(5)} \sim A_Z^{(6)} \sim A_M^{(5)} \sim A_M^{(6)} \sim 0.1\,. \tag{79}$$

Thus, for both scenarios, $k = 5$ and $k = 6$, we find that the axion decay constant is typically constrained to be at most of $\mathcal{O}\left(10^{10}\right)$ GeV. Given this result, it is worthwhile to to recall that, in order to realize gravitino masses of $\mathcal{O}\left(100\right)$ TeV, we anticipate f_a to actually take a value close to $10^{10}\,\mathrm{GeV}$ (see the discussion below Eq. (32)). Our above estimate of the upper bound on f_a is, hence, consistent with this expectation—albeit it seems as if f_a should be rather close to its upper bound in order to allow for the possibility of gravitino masses of $\mathcal{O}\left(100\right)$ TeV. We will specify these statements in the next section, where we will finally present our bounds on f_a along with the corresponding values of $m_{3/2}$. Before we are able to do so, there is, however, one more issue which we need to address. In addition to the upper bounds on f_a derived above, the requirement of perturbative gauge coupling unification at the GUT scale also results in *lower* bounds on f_a.

The new quark flavors affect the running of the SM gauge coupling constants. As we take the new quark fields to transform in complete $SU(5)$ multiplets, i.e., **5** and **5***, the gauge couplings still unify at the same energy scale as in the MSSM, $\Lambda_{\mathrm{GUT}} \simeq 2 \times 10^{16}\,\mathrm{GeV}$. Between the new quark mass scale, m_Q, and the GUT scale, the new quarks, however, contribute to the beta functions of the SM gauge coupling constants, which results in a faster running and, hence, a larger value of the GUT gauge coupling, $\alpha_{\mathrm{GUT}} = g_{\mathrm{GUT}}^2 / \left(4\pi\right)$ than in the MSSM, $\alpha_{\mathrm{GUT}}^{\mathrm{MSSM}} \simeq 1/25$. Thus, in order for our model to be consistent all the way up to the GUT scale, we must require that the SM gauge couplings still unify at some perturbative value, i.e., that α_{GUT} does not exceed unity. The actual value of α_{GUT} in our model depends on the details of the MSSM mass spectrum as well as on the number and the mass scale of the new quark flavors (i.e., k and m_Q). In particular, it increases with k and decreases with m_Q. For a given MSSM mass spectrum and a fixed value of k, the requirement that α_{GUT} must remain perturbative can then be translated into a lower bound on the mass scale of the new quark flavors,[12]

$$\alpha_{\mathrm{GUT}} = \alpha_{\mathrm{GUT}}\left(m_{\mathrm{MSSM}}; k, m_Q\right)\,, \quad \alpha_{\mathrm{GUT}} \leq 1 \;\Rightarrow\; m_Q \geq m_Q^{\mathrm{min}}\left(m_{\mathrm{MSSM}}; k\right)\,. \tag{80}$$

---

[12]While in principle this is a correct statement, in practice, we need to be a bit more careful: Because of the uncertainty in the low-energy input parameters, the uncertainties in the MSSM mass spectrum, etc., the SM gauge couplings do not always unify *exactly* at Λ_{GUT}. Instead, the electroweak gauge couplings often unify, $\alpha_1 = \alpha_2$, before they actually reach the strong gauge coupling α_3. At the technical level, we therefore have to impose the condition that α_3 (and not "α_{GUT}") must remain perturbative, i.e., we require $\alpha_3 \leq 1$ at the scale where α_1 and α_2 unify.
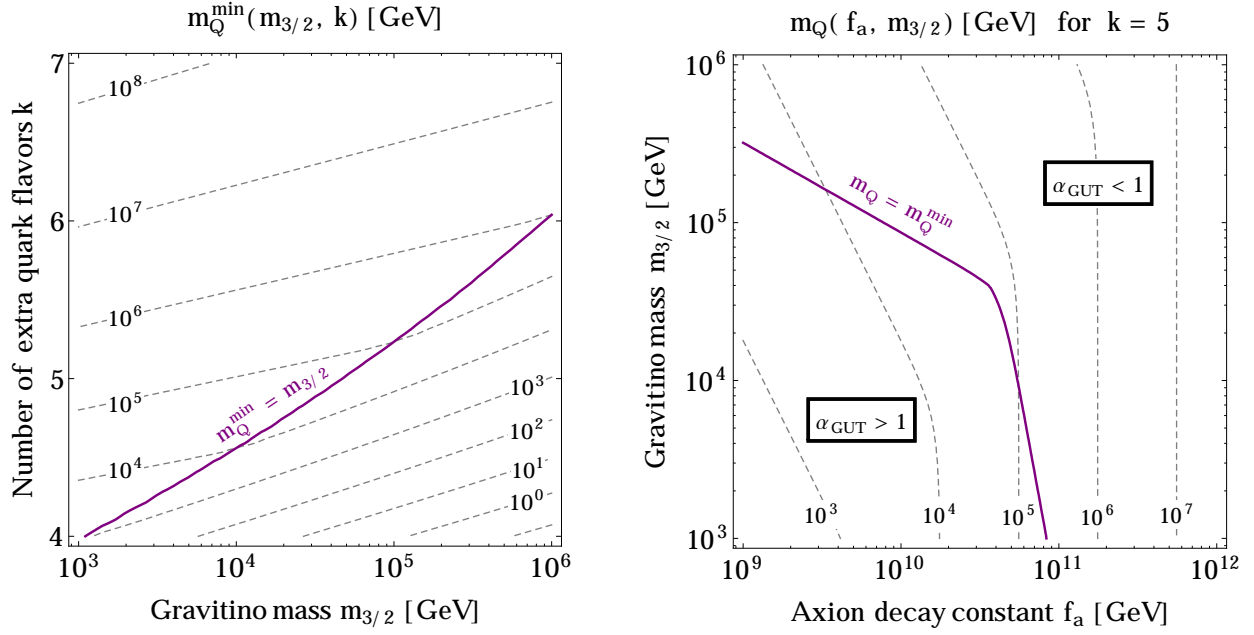
Figure 1: Left panel: Lower bound on the masses of the new quark fields, $m_Q^{\min}$, as function of the gravitino mass and the number of extra quark flavors (see Eqs. (82) and (83)). **Right panel:** New quark mass scale, m_Q, as well as lower bound on the axion decay constant as function of the gravitino mass, $f_Q^{(k)}$, according to the requirement of perturbative gauge coupling unification (i.e., according to the requirement $m_Q \geq m_Q^{\min}$, see Eqs. (87) and (89)) for $k = 5$. Here, all other relevant parameters have been chosen as follows: $\rho = 1$, $\kappa = 1$, $\eta = 4\pi$, $C_{Q_i} = 4\pi$, $M_* = M_{\mathrm{Pl}}$.

In the context of PGM, the MSSM spectrum is basically characterized by two scales: (i) the gravitino mass $m_{3/2}$, which determines the masses of all sfermions as well as of the higgsinos, and (ii) the gaugino mass scale $m_{1/2}$, which is related to the gravitino mass via a loop factor in PGM, $m_{1/2} \sim m_{3/2}/\left(16\pi^2\right)$, and which determines the masses of the MSSM gauginos. Motivated by the perspective of neutralino dark matter, we shall take the gaugino mass scale to be of $\mathcal{O}(1)\,\mathrm{TeV}$ and treat $m_{3/2}$ as a free parameter in the following. Consequently, the lower bound on the new quark mass scale, $m_Q^{\min}$, then becomes a function of $m_{3/2}$ and k only.

In order to determine $m_Q^{\min}$ as a function of these two parameters, we have to solve the renormalization group equations (RGEs) for the SM gauge couplings. We do so numerically and accounting for, in total, three different mass thresholds: We set all gaugino masses to $1\,\mathrm{TeV}$, take the masses of all other MSSM sparticles to be equal to $m_{3/2}$, and assume all new quark flavors to have a common mass equal to m_Q. Between the Z pole and $\max\left\{m_{3/2}, m_Q\right\}$, we simply use the ordinary MSSM one-loop beta functions (including the contributions from the new quark pairs), while for energies between $\max\left\{m_{3/2}, m_Q\right\}$ and Λ_{GUT}, we perform a two-loop calculation in the $\overline{\mathrm{DR}}$ scheme. The idea behind this procedure is that the SM gauge couplings become more sensitive to small changes in the beta functions, the larger they are. We should therefore be a bit more careful in tracking the running of SM gauge couplings at larger energies than at lower energies, which is why we switch from a one-loop analysis to a two-loop analysis beyond the last mass threshold. The result of our calculation is shown in the left panel of Fig. 1, which displays $m_Q^{\min}$ as a function of $m_{3/2}$ and k. Note that, here, k is treated as a continuous parameter, although, of course, only

integer values of k are physically sensible. For $k = 5$ and $k = 6$, for instance, and a gravitino mass of $100 \, \text{TeV}$, we respectively find (setting all other parameters to the same values as in Fig. 1),

$$k = 5 : \quad m_Q^{\min} \simeq 1.8 \times 10^4 \, \text{GeV} \,, \quad k = 6 : \quad m_Q^{\min} \simeq 2.1 \times 10^6 \, \text{GeV} \,. \tag{81}$$

The purple line in the left panel of Fig. 1 indicates the boundary between two different hierarchy schemes that are possible within our set-up. Above the purple line, the new quarks are required to be heavier than the MSSM sfermions; below the purple line, they can also be lighter than the MSSM sfermions. In passing, we also mention that our numerical result for $m_Q^{\min}$ is nicely fit by the following analytical expression,

$$m_Q^{\min} \simeq 10^p \, \text{GeV} \left(\frac{m_{3/2}}{100 \, \text{TeV}} \right)^q \,, \tag{82}$$

where the powers p and q can be expanded into polynomials in $\Delta k = k - 5.2$,

$$p = \begin{cases} 5.0 + 2.9 \, \Delta k - 0.85 \, \Delta k^2 \\ 5.0 + 1.8 \, \Delta k - 0.20 \, \Delta k^2 \end{cases} \,, \quad q = \begin{cases} -1.2 + 0.40 \, \Delta k + 0.11 \, \Delta k^2 & ; \; m_Q^{\min} \lesssim m_{3/2} \\ -0.37 - 0.02 \, \Delta k + 0.03 \, \Delta k^2 & ; \; m_Q^{\min} \gtrsim m_{3/2} \end{cases} \,. \tag{83}$$

Here, $k \simeq 5.2$ corresponds to the k value for which $m_Q^{\min} \simeq m_{3/2} \simeq 10^5 \, \text{GeV}$. We emphasize that this result for $m_Q^{\min}$ holds independently of all other details of our axion model. In fact, it represents nothing but the universal lower bounds on the masses of k pairs of $\mathbf{5}$ and $\mathbf{5}^*$ multiplets imposed by the requirement of perturbative gauge coupling unification for a specific PGM-inspired MSSM mass spectrum. For this reason, we believe that it may also be useful in the context of other scenarios, where the MSSM particle content is supplemented by further $SU(5)$ representations.

For given k and $m_{3/2}$, the constraint on the new quark mass scale in Eq. (82) now implies a lower bound on f_a. To see this, let us rewrite m_Q in Eq. (56) as a function of f_a and ζ. Eqs. (24) and (32) allow us to write the gravitino mass as a function of f_a and ζ first, which leads us to

$$m_{3/2} = \frac{\kappa \, \eta \, k^2 \, \rho^2 \, \zeta^{1/2} \, (2 - \zeta)^{1/2}}{4 \, (1 - \zeta)^{1/2}} \frac{f_a^2}{\sqrt{3} \, M_{\text{Pl}}} \,, \quad m_Q = \frac{C_Q \, k^2 \, \rho^2}{4 \, (1 - \zeta)^{1/2}} \frac{f_a^2}{M_*} \,. \tag{84}$$

Requiring m_Q to be larger than $m_Q^{\min}$ then provides us with the following lower bound on f_a,

$$f_a \gtrsim f_Q^{(k)} = \frac{2 \, (1 - \zeta)^{1/4}}{C_Q^{1/2} \, k \, \rho} \left[m_Q^{\min} \left(m_{3/2}, k \right) M_* \right]^{1/2} \,, \quad m_{3/2} = m_{3/2} \big(f_Q^{(k)}, \zeta \big) \,, \tag{85}$$

with $m_Q^{\min} \left(m_{3/2}, k \right)$ being given in Eqs. (82) and (83) and with $m_{3/2} \left(f_a, \zeta \right)$ being given in Eq. (84). Notice that Eq. (85) only represents an *implicit* definition of our lower bound on the axion decay constant, as $f_Q^{(k)}$ still appears in the argument of the gravitino mass on the right-hand side. In order to evaluate our lower bound on the axion decay constant numerically, it is therefore still necessary, for any given set of input parameter values, to solve Eq. (85) self-consistently for $f_Q^{(k)}$. Alternatively, we may also trade the ζ dependence of $f_Q^{(k)}$ for a dependence on the gravitino mass. To do so, we simply have to use the following relation, which immediately follows from Eq. (84),

$$\zeta = 1 - \left[1 + \left(\frac{4\sqrt{3} \, m_{3/2} \, M_{\text{Pl}}}{\kappa \, \eta \, k^2 \, \rho^2 \, f_a^2} \right)^2 \right]^{-1/2} \,. \tag{86}$$

Plugging this relation into Eq. (85) and solving for $f_Q^{(k)}$, we obtain the following *explicit* expression,

$$f_a \gtrsim f_Q^{(k)} = \left(\frac{4\sqrt{3}\, m_{3/2}\, M_{\mathrm{Pl}}}{\sqrt{2}\, \kappa\, \eta\, k^2\, \rho^2} \right)^{1/2} \left[\left(1 + \left[\frac{\sqrt{2}\, \kappa\, \eta}{C_Q} \frac{m_Q^{\min}\, (m_{3/2}, k)\, M_*}{\sqrt{3}\, m_{3/2}\, M_{\mathrm{Pl}}} \right]^4 \right)^{1/2} - 1 \right]^{1/4} . \tag{87}$$

For $k = 5$ and $k = 6$, for instance, and taking $m_{3/2}$ to be $100\,\mathrm{TeV}$, this bounds evaluates to (again setting all other parameters to the same values as in Fig. 1),

$$k = 5 : \quad f_Q^{(k)} \simeq 7.8 \times 10^9\,\mathrm{GeV}\,, \quad k = 6 : \quad f_Q^{(k)} \simeq 2.1 \times 10^{11}\,\mathrm{GeV}\,. \tag{88}$$

At the same time, Eq. (86) also allows us to rewrite m_Q as a function of f_a and $m_{3/2}$,

$$m_Q = \frac{C_Q\, k^2\, \rho^2}{4} \frac{f_a^2}{M_*} \left[1 + \left(\frac{4\sqrt{3}\, m_{3/2}\, M_{\mathrm{Pl}}}{\kappa\, \eta\, k^2\, \rho^2\, f_a^2} \right)^2 \right]^{1/4} . \tag{89}$$

We plot the expressions for $f_Q^{(k)}$ and m_Q in Eqs. (87) and (89) in the right panel of Fig. 1 for the special case of $k = 5$ extra quark pairs. For $k = 5$ and the values of the gravitino mass that we are most interested in, $m_{3/2} \sim 100\,\mathrm{TeV}$, we again find a bound of $\mathcal{O}\left(10^{10}\right)$ GeV on the axion decay constant—which this time is a lower bound and not an upper bound on f_a. In summary, it therefore seems as if, for $k = 5$, the axion decay must indeed be of $\mathcal{O}\left(10^{10}\right)$ GeV, i.e., it must neither be much smaller nor much larger than $10^{10}\,\mathrm{GeV}$ in order to satisfy all phenomenological constraints at the same time (see also our remarks below Eqs. (32) and (79), respectively). In the next section, we are now going to specify these statements in a bit more detail.

3.3 Final results: viable region in parameter space

The axion decay constant is bounded from above as well as from below (see Eqs. (75) and (85)). Thus, in order to asses the viability of our model, we have to search for regions in parameter space where *not all* possible values of f_a are ruled out, but which still allow for a viable range for f_a,

$$f_Q^{(k)} \lesssim f_a \lesssim f_{\mathrm{PQ}}^{(k)}\,, \quad f_{\mathrm{PQ}}^{(k)} = \min\left\{ f_Z^{(k)}, f_M^{(k)} \right\} . \tag{90}$$

Here, the bounds $f_Q^{(k)}$ and $f_{\mathrm{PQ}}^{(k)}$ are functions of, in total, nine different parameters, which provides us with a lot of freedom when it comes to picking a concrete realization of our axion model. Due to this large parametric freedom, the bounds $f_Q^{(k)}$ and $f_{\mathrm{PQ}}^{(k)}$ can in principle vary over many orders of magnitude, so that it turns out impossible to derive a single *unique* range of values that the axion decay constant is confined to. Also, a systematic scan of the nine-dimensional parameter scan appears to be difficult (and perhaps also not very revealing). Therefore, we will simply focus on certain representative parameter choices in the following, trying to assess what is achievable in our model. To do so, let us first recall which nine parameters $f_Q^{(k)}$ and $f_{\mathrm{PQ}}^{(k)}$ actually depend on:

- The number of extra quark pairs, k (see Eq. (53)). According to our considerations in Sec. 3.2, the integer k must be $k = 5$ or larger (see Eq. (72)). At the same time, increasing the value

of k implies an increase in all three bounds on f_a. For too many extra quark pairs, the lower bound $f_Q^{(k)}$ will therefore begin to exceed the upper boundary of the phenomenologically viable window for the axion decay constant, $f_Q^{(k)} \gtrsim 10^{12}\,\text{GeV}$. For this reason, we will restrict ourselves to scenarios with $k = 5$, $k = 6$ or $k = 7$ extra quark pairs in the following.

- The Yukawa coupling in the IYIT superpotential, λ, or alternatively the parameter $\zeta \in [0, 1]$, which parametrizes the suppression of the meson VEVs $\langle M_\pm \rangle$ (see Eqs. (12) and (15)). Note that ζ can also always be traded for the gravitino mass $m_{3/2}$ via the relation in Eq. (86). In the following, we will mainly be interested in those values of λ (or ζ) that yield a gravitino mass of $100\,\text{TeV}$. This then eliminates the coupling λ as a free parameter from our analysis.

- The parameter $\rho \in [0, 1]$, which represents a measure of the hierarchy among the Yukawa couplings λ_+ and λ_- in the IYIT sector (see Eq. (29)). As evident from Eqs. (75) and (87), all bounds on f_a increase when going to smaller values of ρ. Here, the lower bound $f_Q^{(k)}$ increases, in particular, faster than the upper bound $f_M^{(k)}$. In order to maximize the allowed region in parameter space, we should therefore choose the parameter ρ as large as possible, $\rho = 1$. Interestingly enough, this coincides with the flavor-symmetric limit, $\lambda_+ = \lambda_-$, in the IYIT sector and, hence, might be regarded as a sensible and well motivated choice.

- The parameter κ, which indicates the physical status of the Lagrange multiplier field X. As noted below Eq. (4), κ should be either treated as an $\mathcal{O}(1)$ coupling or sent to infinity. In the former case (i.e., when the field X is assumed to be physical), larger κ values turn out to be more advantageous for our purposes, $\kappa \gtrsim 1$, because going to larger values of κ relaxes both the bounds $f_Q^{(k)}$ and $f_Z^{(k)}$ (the bound $f_M^{(k)}$ is rather insensitive to κ). We will therefore distinguish between three different cases in the following: $\kappa = 1$, $\kappa = 4$, and $\kappa \to \infty$.

- The NDA parameter η, which captures the numerical uncertainty of all coupling constants in the low-energy effective theory induced by strong-coupling effects (see Eq. (4)). Larger η implies a stronger bound on f_a coming from the $M_\pm^k$ meson operators in the superpotential (see Eq. (73) and (76)), which is why we should actually choose η as small as possible, $\eta \simeq \pi$. On the other hand, η is naively expected to be of $\mathcal{O}(4\pi)$, which is why we will consider two cases in the following: $\eta = \pi$ and $\eta = 4\pi$.

- The high-energy cut-off scale M_* in the PQ-breaking operators in $W_{\text{PQ}}^{\text{eff}}$ and $K_{\text{PQ}}^{\text{eff}}$ (see Eq. (48)). As we take these operators to be generated via gravitational interactions, M_* is expected to be close to the Planck scale. For now, we will therefore simply set $M_* = M_{\text{Pl}}$.

- The three dimensionless coefficients C_Q, $C_{Z_\pm}$, and $C_{M_\pm}$ (see Eqs. (54) and (64)), which we expect to take values somewhere between 1 and 4π. To maximize the allowed region in parameter space, the coefficients $C_{Z_\pm}$ and $C_{M_\pm}$ should be chosen as small as possible (see Eq. (75)), while the coefficient C_Q should be chosen as large as possible (see Eq. (85)). We will therefore set $C_{Z_\pm} = C_{M_\pm} = 1$ and $C_Q = 4\pi$ in the following.
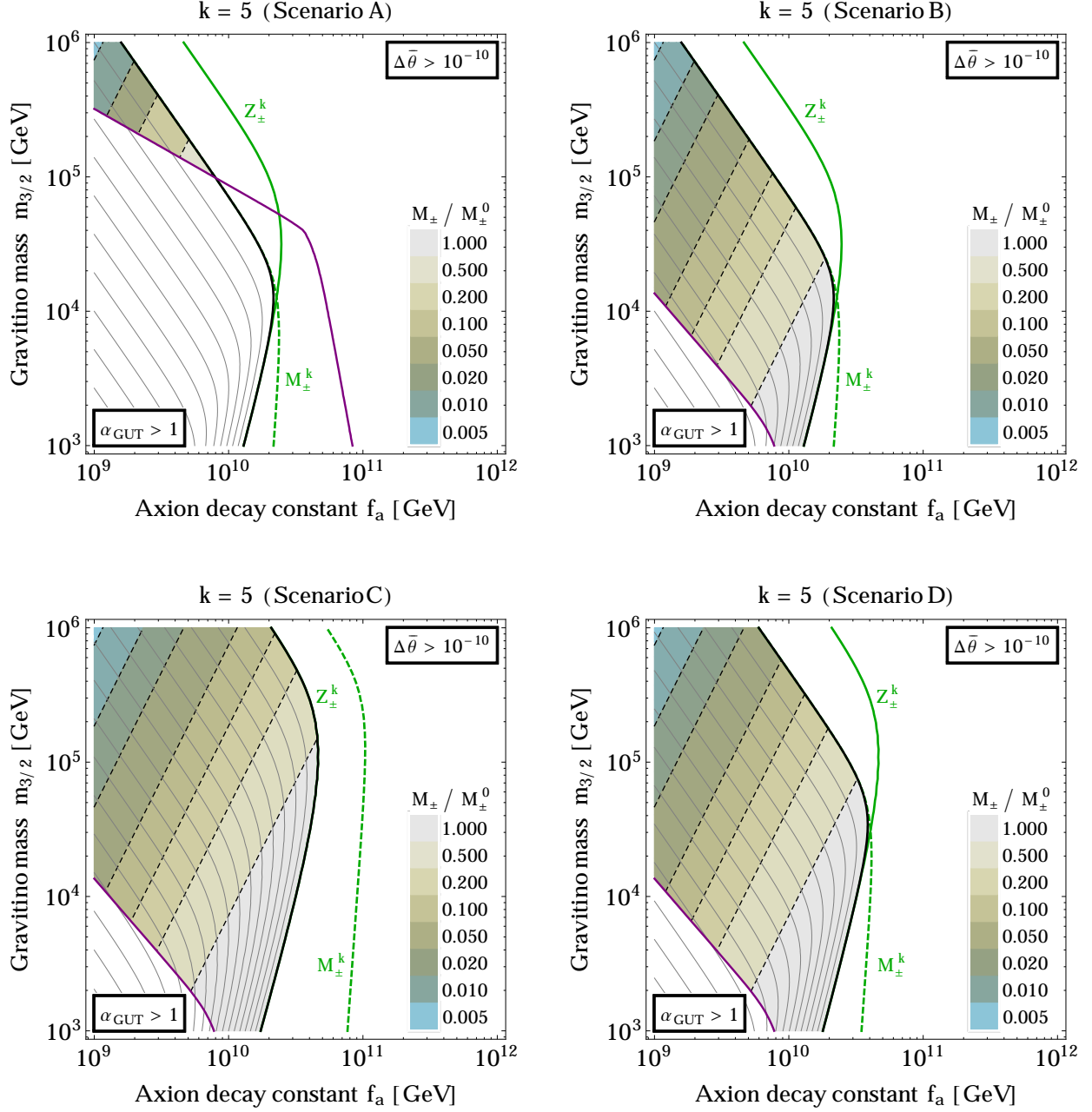
Figure 2: Constraints on the axion decay constant f_a and the gravitino mass $m_{3/2}$ for the four different parameter scenarios specified in Eqs. (91), (93), and (96). The thick solid and dashed green lines represent the upper bounds $f_Z^{(k)}$ and $f_M^{(k)}$ (see Eq. (75)), respectively, while the thick purple lines show the lower bound $f_Q^{(k)}$ (see Eq. (85)). The color code and the dashed black lines indicate the suppression of the meson VEVs compared to the asymptotic expression $M_{\pm}^0$ (see Eq. (16)), i.e., the value of the parameter $\varepsilon = (1 - \zeta)^{1/2}$, where $\zeta = (\lambda/\kappa/\eta)^2$ (see Eq. (15)). Here, large suppression corresponds to a large Yukawa coupling $\lambda \simeq \kappa\eta$, while small suppression corresponds to a small Yukawa coupling $\lambda \ll \kappa\eta$. The thin solid gray lines indicate the value of $\Delta\bar{\theta}$ in integer steps on a logarithmic scale, $\Delta\bar{\theta} = 10^{-11}, 10^{-12}, \cdots$. The thick black solid line marks the values of f_a and $m_{3/2}$ for which $\Delta\bar{\theta} = 10^{-10}$.

Let us now study all combinations of the parameters k, κ, and η according to the above list of restrictions. Remarkably enough, it turns out that there is actually only *one* combination, which happens to allow for a viable range of f_a values for a gravitino mass of $100\,\mathrm{TeV}$!

$$\text{Scenario } A: \qquad k = 5\,, \quad \kappa = 4\,, \quad \eta = \pi \quad \Rightarrow \quad m_{3/2} = 100\,\mathrm{TeV}\,, \quad f_a \simeq 8 \times 10^9\,\mathrm{GeV}\,. \qquad (91)$$

In this scenario (referred to as Scenario A in the following), the axion decay constant ends up being tightly constrained to a value close to $f_a \simeq 10^{10}\,\mathrm{GeV}$ (as expected). In addition, we now see that our axion model turns out to yield a *unique* prediction for the number of extra quark pairs: We have to introduce exactly $k = 5$ pairs of $\mathbf{5}$ and $\mathbf{5}^*$ multiplets—no more, no less. Furthermore, we find that the parameter κ is required to take a finite value. This means that the Lagrange multiplier field X *must* correspond to a dynamical field. Sending κ to infinity (and hence assuming the Lagrange multiplier X to be unphysical) is not an option in Scenario A. Finally, if we allow the gravitino mass to vary, also our constraints on f_a begin to change. This is shown in the upper left panel of Fig. 2, which displays the constraints on f_a and $m_{3/2}$ in the case of Scenario A. As illustrated by this plot, the meson VEVs are always suppressed in Scenario A, $M_\pm/M_\pm^0 \lesssim 0.1$. This indicates that, in Scenario A, the Yukawa coupling λ is required to be rather large.

While f_a and $m_{3/2}$ are found to be tightly constrained in the minimal version of our model (i.e, in Scenario A), there are several (almost trivial) possibilities to modify our model, so as to relax the bounds on parameter space. For instance, we may assume a different mechanism to generate the masses of the new quark pairs than in Eq. (54). So far, we have taken the new quark pairs to be coupled to the SUSY-breaking sector via gravitational interactions, i.e., we have taken the high-energy cut-off scale in Eq. (54) to be the scale $M_* \sim M_{\mathrm{Pl}}$. This, however, does not necessarily need to be the case. The new quark pairs might also couple to the SUSY-breaking sector via the exchange of GUT messenger fields Ψ' and $\bar{\Psi}'$ with masses of $\mathcal{O}\left(\Lambda_{\mathrm{GUT}}\right)$ and transforming as fundamentals of both the strongly coupled $SU(2)$ as well as of $SU(5)$,

$$W^Q \supset Q_i \bar{\Psi}' \Psi^1 + \bar{Q}_i \Psi' \Psi^2 + M' \Psi' \bar{\Psi}' \quad \Rightarrow \quad W^Q_{\mathrm{eff}} \supset \frac{1}{M'} \frac{\Lambda}{\eta} \left(Q\bar{Q}\right)_i M_+\,, \quad M' \sim \Lambda_{\mathrm{GUT}}\,. \qquad (92)$$

In this case, the cut-off scale in the effective quark superpotential W^Q_{eff} is no longer of $\mathcal{O}\left(M_{\mathrm{Pl}}\right)$, but rather of $\mathcal{O}\left(\Lambda_{\mathrm{GUT}}\right)$. Effectively, such a situation can be accounted for in our analysis by increasing the coefficient C_Q in Eq. (54) by a factor $M_{\mathrm{Pl}}/\Lambda_{\mathrm{GUT}} \sim 100$. Setting C_Q to $C_Q = 100 \times 4\pi$ then significantly widens the allowed region in parameter space (see the upper right panel of Fig. (2)). We shall refer to this scenario as Scenario B,

$$\text{Scenario } B: \qquad k = 5\,, \quad \kappa = 4\,, \quad \eta = \pi\,, \quad C_Q = 100 \times 4\pi\,. \qquad (93)$$

For $m_{3/2} = 100\,\mathrm{TeV}$, increasing C_Q to such a large value basically removes the lower bound on f_a coming from the requirement of perturbative gauge coupling unification. The axion decay constant then ends up being constrained by the astrophysical bound $f_a \gtrsim 10^9\,\mathrm{GeV}$ as well as by $f^{(k)}_{\mathrm{PQ}}$,

$$m_{3/2} = 100\,\mathrm{TeV} \quad \Rightarrow \quad 10^9\,\mathrm{GeV} \lesssim f_a \lesssim 8 \times 10^9\,\mathrm{GeV}\,. \qquad (94)$$

Here, smaller values of f_a require the meson VEVs $\langle M_\pm \rangle$ to be increasingly suppressed compared to the asymptotic expression $M_\pm^0$ (see Eq. (15)).[13] Too strong a suppression, however, appears implausible, both from the standpoint of our explicit calculation as well as according to our general expectation regarding the behavior of the strongly coupled IYIT sector at low energies. We therefore believe that the axion decay constant has, in general, a tendency of being as large as possible, so as to reduce the suppression of the meson VEVs. As for Scenario B, this means that, despite the significant relaxation of the lower bound $f_Q^{(k)}$, we actually still expect f_a to be of $\mathcal{O}\left(10^{10}\right)$ GeV.

Another trivial possibility to relax the bounds on parameter space is to increase the cut-off scale M_* by some factor of $\mathcal{O}(1 \cdots 4\pi)$.[14] If we do so starting with Scenario A, it becomes difficult to realize gravitino masses of $100\,\mathrm{TeV}$ because $f_Q^{(k)}$ increases too drastically. On the other hand, combining Scenario B with a larger cut-off scale does provide us with a viable scenario that also admits a gravitino mass of $100\,\mathrm{TeV}$. In this case, also κ and η can again be set to different values,

$$\text{Scenario } C: \quad k = 5\,, \quad \kappa = 4\,, \quad \eta = \pi\,, \quad C_Q = 100 \times (4\pi)^2\,, \quad M_* = 4\pi\, M_{\mathrm{Pl}}\,, \quad (95)$$
$$\text{Scenario } D: \quad k = 5\,, \quad \kappa = 1\,, \quad \eta = 4\pi\,, \quad C_Q = 100 \times (4\pi)^2\,, \quad M_* = 4\pi\, M_{\mathrm{Pl}}\,.$$

Here, we have multiplied C_Q by another factor of 4π to keep the ratio C_Q/M_* fixed at the same value as in Scenario B. The bounds on the f_a–$m_{3/2}$ parameter space for these two scenarios are shown in the two lower panels of Fig. 2. For $m_{3/2} = 100\,\mathrm{TeV}$, the axion decay constant is again bounded by the lower astrophysical bound, $f_a \gtrsim 10^9\,\mathrm{GeV}$, in these two scenarios. At the same time, the upper bound on f_a now increases by roughly half an order of magnitude,

$$\text{Scenario } C: \quad f_a \lesssim 5 \times 10^{10}\,\mathrm{GeV}\,, \quad \text{Scenario } D: \quad f_a \lesssim 3 \times 10^{10}\,\mathrm{GeV}\,. \quad (96)$$

Guided by the notion that too strong a suppression of the meson VEVs tends to be unrealistic, we suppose that also in Scenarios C and D the axion decay constant most likely takes a value close to the upper end of the allowed range. That is, once again, we expect $f_a \sim 10^{10}\,\mathrm{GeV}$.

Finally, we mention that, relaxing our restrictions on C_Q and M_* similarly as in Eq. (96), i.e., assuming the new quark masses to be generated at the GUT scale and slightly raising the cut-off scale M_* above the reduced Planck mass M_{Pl}, a number of further interesting scenarios become available. For instance, scenarios with $k > 5$ extra quark pairs now become viable, such as

$$\text{Scenario } E: \quad k = 6\,, \quad \kappa = 1\,, \quad \eta = 4\pi\,, \quad C_Q = 100 \times (4\pi)^2\,, \quad M_* = 4\pi\, M_{\mathrm{Pl}}\,, \quad (97)$$

in the case of which the axion decay constant is again required to take a value of $\mathcal{O}\left(10^{10}\right)$ GeV,

$$m_{3/2} = 100\,\mathrm{TeV} \quad \Rightarrow \quad 7 \times 10^9\,\mathrm{GeV} \lesssim f_a \simeq 5 \times 10^{10}\,\mathrm{GeV}\,. \quad (98)$$

---

[13]Recall that, in our formal calculation (employing a canonical Kähler potential), this is achieved by fine-tuning the Yukawa coupling λ, so that it increasingly approaches its maximal value $\lambda_{\mathrm{max}} = \kappa\,\eta = 4\pi$ (see Eq. (15)).

[14]This may also be desirable from the perspective of flavor-changing neutral currents, which may still be a little bit too large for $m_{3/2} = 100\,\mathrm{TeV}$. Slightly increasing the cut-off scale M_* can then help to *fully* solve the FCNC problem also for a gravitino mass of $100\,\mathrm{TeV}$, i.e., without the need for going to $m_{3/2}$ as large as, say, $1000\,\mathrm{TeV}$ [32, 66].

Furthermore, we may now also assume that the field X is unphysical and send κ to infinity,

$$\text{Scenario } F: \quad k = 6 \,, \quad \kappa \to \infty \,, \quad \eta = 4\pi \,, \quad C_Q = 100 \times (4\pi)^2 \,, \quad M_* = 4\pi\, M_{\text{Pl}} \,. \quad (99)$$

In this scenario, the axion decay constant is then more or less constrained to a certain value,

$$m_{3/2} = 100\,\text{TeV} \quad \Rightarrow \quad f_a \simeq 5 \times 10^{10}\,\text{GeV} \,. \quad (100)$$

A more systematic study of these (and possibly other) scenarios is left for future work. For now, we merely conclude by observing that our model indeed appears to be compatible with all bounds in large parts of parameter space. Without any further assumptions, the number of extra pairs is fixed to be $k = 5$, while the decay constant f_a is generally expected to take a value of $\mathcal{O}\left(10^{10}\right)$ GeV.

4 Conclusions and Outlook

In this paper, we have demonstrated how the PQ solution to the strong CP problem might be inherently connected to the dynamics of spontaneous SUSY breaking. To give a concrete example of our idea, we have embedded the PQ mechanism into the IYIT model of dynamical SUSY breaking (i.e. into a strongly coupled $SU(2)$ gauge theory with four matter and six singlet fields), which has led us to a particular supersymmetric variant of the KSVZ axion model. As a direct consequence of this embedding, we found that the scale of PQ symmetry breaking, Λ_{PQ}, is no longer an arbitrary (and somewhat mysterious) input parameter, but rather directly tied to the dynamical scale Λ of the strong interactions in the SUSY-breaking sector, $\Lambda_{\text{PQ}} \sim \Lambda$. As the same dynamical scale also determines the scale of SUSY breaking in the IYIT model, $\Lambda_{\text{SUSY}} \sim \Lambda$, a PQ scale of $\mathcal{O}\left(10^{11} \cdots 10^{12}\right)$ GeV then implies a large SUSY breaking scale and, hence, a large gravitino mass, $m_{3/2} \sim 100\,\text{TeV}$. The proposed connection between the dynamics of PQ symmetry and SUSY breaking therefore turns out to go very well with the idea of pure gravity mediation.

Besides that, the notion of pure gravity mediation is also crucial to our axion model for another reason: In order to protect the PQ symmetry from the dangerous effect of higher-dimensional operators induced by gravitational interactions around the Planck scale, one has to invoke a protective gauge symmetry—for instance, as proposed in [34], a discrete R symmetry. Among all possible Z_N^R symmetries, pure gravity mediation singles out the special case of a Z_4^R symmetry, which is the only discrete R symmetry that allows to generate the MSSM μ term via a Higgs bilinear term in the Kähler potential. As we were able to show, such a discrete Z_4^R symmetry then manages to suppress all PQ-breaking operators in the superpotential and Kähler potential up to a high order, thereby ensuring that the PQ symmetry is of sufficiently good quality. Solely within the MSSM, however, a discrete Z_4^R symmetry does not represent a good symmetry, as it is anomalously violated at the quantum level by $SU(2)_L$ and $SU(3)_C$ instanton effects. Therefore, in order to render the Z_4^R symmetry anomaly-free, the presence of further SM-charged is required.[15] For this reason,

---

[15]By contrast, solely within the IYIT sector, the Z_4^R *is* anomaly-free and even preserved in the true vacuum. It is, therefore, neither broken explicitly nor spontaneously by the strong interactions, which retains R symmetry as a useful tool to study the low-energy dynamics of the SUSY-breaking sector (see our discussion related to Eq. (57)).

we have assumed the existence of k new quark/antiquark pairs $(Q_i, \bar{Q}_i) \sim (\mathbf{5}, \mathbf{5}^*)$ in this paper, which obtain masses of the order of the gravitino mass from coupling to the SUSY-breaking sector. Provided an appropriate R charge, these new quark fields then cancel the Z_4^R anomalies, which puts us in the position to employ the Z_4^R symmetry as a protective gauge symmetry after all.

The axion model presented in this paper comes with a number of attractive conceptional and phenomenological implications. The extra matter fields, for instance, contribute to the running of the SM gauge couplings, which increases the value of the GUT gauge coupling constant. Requiring that the SM gauge couplings should unify at a perturbative value therefore puts a lower bound on the mass scale of the new quark fields. For one thing, this constrains the parameter space of our model (i.e., it provides us with a lower bound on the axion decay constant). For another, we note that the effect of several new $SU(5)$ multiplets with masses $m_Q \sim m_{3/2}$ on the running of the SM gauge couplings might also play the role of a selection criterion in the landscape of string vacua. Of course, such an assertion is highly speculative; but we have the feeling that it is worth being pointed out nonetheless. Imagine, for instance, that the SM gauge couplings are bound to unify at some $\mathcal{O}(1)$ value at the GUT scale. The fact that the new quark fields obtain their masses via couplings to the SUSY-breaking sector may then potentially bias the distribution of different values of the SUSY breaking scale—maybe the SUSY breaking scale happens to be very large, so that $m_{3/2} \sim 100\,\text{TeV}$, simply because otherwise the SM gauge couplings would run over too long a distance between the GUT scale and the new quark mass threshold. This would then alter the ratios of the SM gauge couplings at the electroweak scale and, for one reason for another (in the context of nuclear and/or atomic physics), maybe exclude the possibility of habitable universes.

Apart from this perhaps far-fetched speculation, our model also makes a number of predictions which are testable in present-day or near-future experiments. First of all, the new quark fields may, for instance, be directly detectable in a multi-TeV collider experiment. In this context, it is interesting to remark that our model (at least in its simplest form) surprisingly singles out a unique number of extra quark pairs: We have to add exactly five pairs of new matter fields. Remarkably enough, this leads to a situation where the R charges of the new quark fields, the R charges of the fields in the SUSY-breaking sector as well as the R charges of the MSSM fields all look very similar. For $k = 5$ (and only for $k = 5$), the R charges of all fields in our model turn out to be multiples of $1/5$. Whether or not this points at something deep remains to be seen; but it is certainly an interesting observation. Moreover, we find that a sufficient suppression of all PQ-breaking effects typically requires the axion decay constant to take a value not much larger than $f_a \sim 10^{10}\,\text{GeV}$. This has several interesting implications for cosmology. To begin with, let us remark that, in our model, the PQ symmetry should be broken before the end of inflation, i.e., the PQ scale needs to exceed the inflationary Hubble scale, $\Lambda_{\text{PQ}} \gtrsim H_{\text{inf}}$. If this was not the case, dangerous axion domain walls (with domain wall number $N_{\text{DW}} = |\mathcal{A}_{\text{PQ}}| = k > 1$) would form during the QCD phase transition, dominating the energy density of the universe soon after their production [67]. However, if the PQ symmetry is already broken during inflation, we have to pay attention that the isocurvature perturbations induced by the axion fluctuation during inflation, $\delta\bar{\theta} \simeq H_{\text{inf}}/(2\pi)$,

do not violate any of the stringent bounds derived from the precise observations of the cosmic microwave background (CMB) [60]. For $f_a \sim 10^{10}\,\mathrm{GeV}$ and an initial axion misalignment angle $\bar{\theta}$ of $\mathcal{O}(1)$, this constrains the Hubble rate during inflation to a rather small value, $H_{\mathrm{inf}} \lesssim 10^8\,\mathrm{GeV}$. Our axion model is therefore only compatible with small-field models of inflation. Or put differently, from the perspective of our model, we are led to expect that upcoming CMB experiments will unfortunately not be able to see any signs of tensor perturbations in the CMB. That is, if the inflationary Hubble rate should indeed be as small as $10^8\,\mathrm{GeV}$, or even smaller, the tensor-to-ratio is at most of $\mathcal{O}(10^{-13})$, which is unfortunately out of reach for any planned CMB experiment. Besides that, an axion decay constant of $\mathcal{O}(10^{10})\,\mathrm{GeV}$ (in combination with $\bar{\theta} \sim 1$ and $\delta\bar{\theta} \ll 1$), results in an axionic contribution to the relic density of dark matter of about $\mathcal{O}(10\,\%)$ [68]. The remaining DM density is then accounted for by weakly interacting massive particles (WIMPs) in the form of MSSM neutralinos in our model. For this reason, we are confident that both axion as well as WIMP dark matter searches may, in principle, be able to find positive signals. Moreover, for $f_a \sim 10^{10}\,\mathrm{GeV}$, the axion mass lies in the meV range. Such relatively heavy axions could, for instance, be searched for in fifth-force experiments searching for axion-mediated long range forces [69] or in experiments aiming at measuring the proton electric dipole moment [70]. Given the fact that $\Delta\bar{\theta}$ may easily take a value only slightly below the upper bound $\Delta\bar{\theta}^{\mathrm{max}}$ in our model, $\Delta\bar{\theta} \sim 10^{-11} \cdots 10^{-12}$, (see the upper left panel of Fig. 2) such experiments look indeed promising.

In summary, we therefore conclude that our axion model not only appears to provide an interesting link between dynamical SUSY breaking and the PQ mechanism, it also gives rise to a rich phenomenology that is going to be tested in current and upcoming experiments. This is exciting and hopefully only a first step towards a better understanding of supersymmetry, dark matter and the new physics lurking behind the strong CP problem—which, as we believe, should certainly star some kind of *axion field of dynamical origin* as the main protagonist.

A   Exact vacuum of the IYIT model for canonical Kähler potential

In this appendix, we compute the exact VEVs of the SUSY-breaking singlet fields in the IYIT model, Z_+, Z_-, and X, under the simplifying assumption of a canonical Kähler potential for all relevant fields. Of course, this assumption can never hold true *exactly*, as, in general, strong-coupling effects will always induce higher-dimensional terms in the effective Kähler potential. Still,

we deem a calculation based on a canonical Kähler potential useful for several reasons. First of all, we expect it to represent an important benchmark scenario for the more general case—a benchmark scenario that we have well under control and that allows us to obtain a better understanding of the various parameter dependences in our model. Moreover, since we lack the ability to calculate the dynamical corrections to the Kähler potential, there are, in fact, not many alternatives to assuming a canonical Kähler potential, if we are interested in more than just some rough order-of-magnitude estimates. Starting from a canonical Kähler potential, we are able to derive a consistent set of expressions in terms of a number of well-defined parameters. This would, by contrast, not be possible, if we also intended to account for the uncertainties related to the effective Kähler potential. On top of that, in case the dynamically generated corrections to the Kähler potential become smaller and smaller, the expressions that we are going to derive in the following become increasingly accurate, approximating the true results in the IYIT model with arbitrary precision. All in all, we therefore believe that the simplifying assumption of a canonical Kähler potential—while not exactly reflecting the actual situation in the IYIT model—still captures many of the aspects that we are interested in and that it is, hence, worth a closer examination.

The starting point of our analysis is the effective superpotential in Eq. (12) in combination with a canonical Kähler potential for all DOFs in the low-energy effective theory,

$$W_{\text{eff}} \simeq \kappa \, \eta \, X \left[\text{Pf} \left(M^{ij} \right) - \left(\frac{\Lambda}{\eta} \right)^2 \right] + \frac{\Lambda}{\eta} \left(\lambda_+ \, M_+ \, Z_- + \lambda_- \, M_- \, Z_+ + \lambda_0^a \, M_0^a \, Z_0^a \right) , \qquad (101)$$

$$K_{\text{eff}} \simeq |X|^2 + |Z_+|^2 + |Z_-|^2 + |M_+|^2 + |M_-|^2 + \sum_{a=1}^{4} |Z_0^a|^2 + \sum_{a=1}^{4} |M_0^a|^2 .$$

In the true vacuum of the scalar potential corresponding to these input functions (assuming $\lambda_+ \lambda_-$ to be the smallest among the three products $\lambda_+ \lambda_-$, $\lambda_0^1 \lambda_0^4$, and $\lambda_0^2 \lambda_0^3$), SUSY is broken by the F-term of the following linear combination of the fields Z_+, Z_-, and X (see Eqs. (22) and (34)),

$$S_0 = \frac{1}{(2-\zeta)^{1/2}} \left[(1-\zeta)^{1/2} \left(Z_+ + Z_- \right) - \zeta^{1/2} X \right] , \quad |F_{S_0}| = \mu^2 = \lambda \, (2-\zeta)^{1/2} \, \frac{\Lambda^2}{\eta^2} . \qquad (102)$$

Taking into account the spontaneous breaking of R symmetry in the context of SUGRA, this linear combination turns out to acquire a nonzero VEV of $\mathcal{O}\left(m_{3/2} \right)$. To see this, first of all note that R symmetry breaking induces a constant term W_0 in the superpotential (see Eq. (23)),

$$W \supset W_0 = m_{3/2} \, M_{\text{Pl}}^2 . \qquad (103)$$

Together with the SUSY-breaking tadpole term for the goldstino field S_0 in the effective superpotential, $W_{\text{eff}} \supset \mu^2 S_0$ (see Eq. (38)), and together with the loop-induced mass for the complex sgoldstino $s_0 \subset S_0$ in the effective scalar potential, $V_{\text{eff}} \supset m_{s_0}^2 \, |s_0|^2$ (see Eq. (41)), this constant superpotential gives rise to the following total scalar potential for the complex scalar s_0,

$$V_{\text{eff}} = m_{s_0}^2 \, |s_0|^2 - 2 \, m_{3/2} \, \mu^2 \left(s_0 + s_0^* \right) . \qquad (104)$$

We, thus, find that the interplay between the constant superpotential W_0 and the tadpole term $\mu^2 S_0$ breaks the rotational invariance in the complex s_0 plane. That is, while the imaginary part of s_0 remains stabilized at 0 thanks to the loop-induced mass m_{s_0}, the real component of s_0 obtains a linear potential proportional to $m_{3/2}\,\mu^2$, which shifts its VEV from 0 to some value of $\mathcal{O}\left(m_{3/2}\right)$,

$$\langle \mathrm{Re}\left\{s_0\right\}\rangle = \frac{2\,\mu^2}{m_{s_0}^2}\,m_{3/2}\,, \quad \langle \mathrm{Im}\left\{s_0\right\}\rangle = 0 \quad \Rightarrow \quad \langle S_0 \rangle = \frac{2\,\mu^2}{m_{s_0}^2}\,m_{3/2}\,. \tag{105}$$

This result readily translates into expressions for $\langle Z_+ \rangle$, $\langle Z_- \rangle$, and $\langle X \rangle$. All we need to know is the inverse of the transformation between the two field bases (Z_+, Z_-, X) and (S_0, S_1, S_2) in Eq. (34),

$$Z_\pm = \frac{1}{(2-\zeta)^{1/2}}\left[(1-\zeta)^{1/2}\,S_0 \pm 2^{-1/2}\,(2-\zeta)^{1/2}\,S_1 + (\zeta/2)^{1/2}\,S_2\right]\,, \tag{106}$$

$$X = \frac{1}{(2-\zeta)^{1/2}}\left[-\zeta^{1/2}S_0 + 2^{1/2}\,(1-\zeta)^{1/2}\,S_2\right]\,.$$

Taking into account that the singlets S_1 and S_2 do *not* obtain a nonzero VEV, this leads us to

$$\langle Z_\pm \rangle = \left(\frac{1-\zeta}{2-\zeta}\right)^{1/2}\langle S_0 \rangle = \frac{1}{\sqrt{2}}\left(1-r^2\right)^{1/2}\langle S_0 \rangle\,, \quad \langle X \rangle = -\left(\frac{\zeta}{2-\zeta}\right)^{1/2}\langle S_0 \rangle = -r\,\langle S_0 \rangle\,, \tag{107}$$

where we have used Eq. (36) to rewrite the ζ-dependent coefficients in terms of the parameter r. From Eqs. (105) and (107), we now see that all singlet VEVs crucially depend on the loop-induced sgoldstino mass m_{s_0}. In order to obtain usable expressions for $\langle Z_+ \rangle$, $\langle Z_- \rangle$, and $\langle X \rangle$, we therefore need to determine this mass parameter as precisely as possible. This is what we shall do next.

The sgoldstino mass m_{s_0} follows from the one-loop effective Coleman-Weinberg potential,

$$m_{s_0}^2 = \left.\frac{\partial^2 V_{\mathrm{CW}}}{\partial s_0\,\partial s_0^*}\right|_{s_0=0}\,, \quad V_{\mathrm{CW}} = \frac{1}{64\pi^2}\,\mathrm{STr}\left[M^4\left(\ln\left(\frac{M^2}{Q^2}\right)+c\right)\right]\,, \tag{108}$$

where M^2 stands for the total mass matrix of the IYIT model squared, Q denotes an appropriate renormalization scale for the low-energy effective theory and c is a constant that is sometimes introduced for cosmetic reasons, but which may as well also be simply absorbed into the scale Q. In order to evaluate V_{CW} and determine m_{s_0}, we therefore need to compute the entire mass spectrum of the IYIT model for a nonzero value of the goldstino field S_0. In the charged meson sector (which includes the chiral superfields $M_\pm$, $Z_\pm$, and X), the physical mass eigenstates correspond to ten real scalars, one Weyl fermion and four Majorana fermions (see also our discussion at the end of Sec. 2.4). Here, the bosonic DOFs consist of the axion a, the saxion ϕ as well as the real and imaginary parts of the complex scalars contained in the goldstino field, $s_0^\pm$, the singlet field S_1 (which shares a Dirac mass with the axion field), $s_1^\pm$, the radial meson field, $m^\pm$, and the singlet field S_2 (which shares a Dirac mass with the radial meson field), $s_2^\pm$. Meanwhile, the fermionic DOFs consist of the goldstino $\tilde{s}_0$ as well as four Majorana fermions forming two pairs of quasi-Dirac fermions, $(\tilde{a}, \tilde{s}_1)$ and $(\tilde{m}, \tilde{s}_2)$, where $\tilde{a}$ stands for the axino. A straightforward calculation of the

bosonic mass matrix in global SUSY and at tree level, accounting for nonzero S_0, then yields

$$m_\phi^2 = m^2 \left[\frac{3}{2} + \frac{m^2}{2\,\mu^4} |S_0|^2 + \frac{1}{2} \left(1 + 6\,\frac{m^2}{\mu^4} |S_0|^2 + \frac{m^4}{\mu^8} |S_0|^4 \right)^{1/2} \right], \quad m_a^2 = m_{s_0^\pm}^2 = 0, \qquad (109)$$

$$m_{s_1^-}^2 = m^2 \left[\frac{3}{2} + \frac{m^2}{2\,\mu^4} |S_0|^2 - \frac{1}{2} \left(1 + 6\,\frac{m^2}{\mu^4} |S_0|^2 + \frac{m^4}{\mu^8} |S_0|^4 \right)^{1/2} \right], \quad m_{s_1^+}^2 = m^2 + \frac{m^2}{\mu^4} |S_0|^2,$$

$$m_{m^\pm}^2 = \frac{m^2}{r^2} \left[1 \pm \frac{r^2}{2} + \frac{r^2 m^2}{2\,\mu^4} |S_0|^2 \pm \frac{r^2}{2} \left(1 + \frac{2}{r^2} \left(2 \pm r^2 \right) \frac{m^2}{\mu^4} |S_0|^2 + \frac{m^4}{\mu^8} |S_0|^4 \right)^{1/2} \right],$$

$$m_{s_2^\pm}^2 = \frac{m^2}{r^2} \left[1 \pm \frac{r^2}{2} + \frac{r^2 m^2}{2\,\mu^4} |S_0|^2 \mp \frac{r^2}{2} \left(1 + \frac{2}{r^2} \left(2 \pm r^2 \right) \frac{m^2}{\mu^4} |S_0|^2 + \frac{m^4}{\mu^8} |S_0|^4 \right)^{1/2} \right],$$

while a similar calculation of the fermionic mass matrix provides us with

$$m_{(\tilde{a},\tilde{s}_1)}^2 = m^2 \left[1 + \frac{m^2}{2\,\mu^4} |S_0|^2 \pm \left(\frac{m^2}{\mu^4} |S_0|^2 + \frac{m^4}{4\,\mu^8} |S_0|^4 \right)^{1/2} \right], \quad m_{\tilde{s}_0}^2 = 0, \qquad (110)$$

$$m_{(\tilde{m},\tilde{s}_2)}^2 = \frac{m^2}{r^2} \left[1 + \frac{r^2 m^2}{2\,\mu^4} |S_0|^2 \pm r^2 \left(\frac{1}{r^2} \frac{m^2}{\mu^4} |S_0|^2 + \frac{m^4}{4\,\mu^8} |S_0|^4 \right)^{1/2} \right].$$

At the same time, the neutral meson sector (which includes the chiral superfields M_0^a and Z_0^a, where $a = 1, 2, 3, 4$) also features S_0-dependent mass eigenvalues. The reason for this is the coupling of the field $X = -r\,S_0 + \cdots$ to the Pfaffian of the complete meson matrix in Eq. (12). In fact, the total effective superpotential for the neutral meson fields takes the following form,

$$W_{\text{eff}} \supset \kappa\,\eta \left[r\,S_0 - \left(1 - r^2 \right)^{1/2} S_2 \right] \left(M_0^1 M_0^4 - M_0^2 M_0^3 \right) + \lambda_0^a\,M_0^a\,Z_0^a, \qquad (111)$$

which gives rise to eight complex scalars as well as to four Dirac fermions. Here, the masses of the four complex scalars, $m_{14}^\pm$ and $m_{23}^\pm$, contained in the neutral meson fields M_0^a are gives as

$$m_{m_{14}^\pm}^2 = \left(\sigma_{14}^2 \pm \lambda_{14}^2 \right) \left(\frac{\Lambda}{\eta} \right)^2 \left[1 \pm \frac{\kappa^2\,\eta^2\,r^2}{\lambda_{14}^2\,\Lambda^2/\eta^2} |S_0|^2 + \mathcal{O} \left(|S_0|^4 \right) \right], \qquad (112)$$

$$m_{m_{23}^\pm}^2 = \left(\sigma_{23}^2 \pm \lambda_{23}^2 \right) \left(\frac{\Lambda}{\eta} \right)^2 \left[1 \pm \frac{\kappa^2\,\eta^2\,r^2}{\lambda_{23}^2\,\Lambda^2/\eta^2} |S_0|^2 + \mathcal{O} \left(|S_0|^4 \right) \right],$$

where we have introduced the symbols σ_{14}, λ_{14}, σ_{23}, and λ_{23}, for the ease of notation (see Eq. (40)),

$$\sigma_{14} = \left[\frac{1}{2} \left(\left(\lambda_0^1 \right)^2 + \left(\lambda_0^4 \right)^2 \right) \right]^{1/2}, \quad \lambda_{14} = \left(\lambda^4 + \delta_{14}^4 \right)^{1/4}, \quad \delta_{14} = \left[\frac{1}{2} \left(\left(\lambda_0^1 \right)^2 - \left(\lambda_0^4 \right)^2 \right) \right]^{1/2}, \quad (113)$$

and similarly for σ_{23}, λ_{23}, and δ_{23}. By contrast, the masses of the complex scalars z_0^a contained in the neutral singlet fields Z_0^a turn out to be independent of S_0 up to corrections of $\mathcal{O}\left(|S_0|^4 \right)$,

$$m_{z_0^a}^2 = \left(\lambda_0^a \right)^2 \left(\frac{\Lambda}{\eta} \right)^2 + \mathcal{O} \left(|S_0|^4 \right), \quad a = 1, 2, 3, 4. \qquad (114)$$

Last but not least, the masses of the four Dirac fermions in the neutral meson sector are given as

$$m^2_{(\tilde{m}_0^1, \tilde{z}_0^1)} = \left(\lambda_0^1\right)^2 \left(\frac{\Lambda}{\eta}\right)^2 \left[1 + \frac{1}{2} \frac{\kappa^2 \eta^2 r^2}{\delta_{14}^2 \Lambda^2/\eta^2} |S_0|^2 + \mathcal{O}\left(|S_0|^4\right)\right], \tag{115}$$

$$m^2_{(\tilde{m}_0^2, \tilde{z}_0^2)} = \left(\lambda_0^2\right)^2 \left(\frac{\Lambda}{\eta}\right)^2 \left[1 + \frac{1}{2} \frac{\kappa^2 \eta^2 r^2}{\delta_{23}^2 \Lambda^2/\eta^2} |S_0|^2 + \mathcal{O}\left(|S_0|^4\right)\right],$$

$$m^2_{(\tilde{m}_0^3, \tilde{z}_0^3)} = \left(\lambda_0^3\right)^2 \left(\frac{\Lambda}{\eta}\right)^2 \left[1 - \frac{1}{2} \frac{\kappa^2 \eta^2 r^2}{\delta_{23}^2 \Lambda^2/\eta^2} |S_0|^2 + \mathcal{O}\left(|S_0|^4\right)\right],$$

$$m^2_{(\tilde{m}_0^4, \tilde{z}_0^4)} = \left(\lambda_0^4\right)^2 \left(\frac{\Lambda}{\eta}\right)^2 \left[1 - \frac{1}{2} \frac{\kappa^2 \eta^2 r^2}{\delta_{14}^2 \Lambda^2/\eta^2} |S_0|^2 + \mathcal{O}\left(|S_0|^4\right)\right].$$

With the expressions in Eqs. (109), (110), (112), (114), and (115) at our disposal, we now know the entire mass spectrum of the IYIT model up to $\mathcal{O}\left(|S_0|^2\right)$. This allows us to evaluate the Coleman-Weinberg potential and, hence, determine the sgoldstino mass. Differentiating V_{CW} w.r.t. to s_0 and s_0^* finally leads us to the following result for m_{s_0} (see Eq. (108)),[16]

$$m^2_{s_0} = \frac{2\ln 2 - 1}{16\pi^2} \left[1 + \omega(r) + \frac{2}{\rho^6} \left(\left(\frac{\lambda_{14}}{\lambda}\right)^2 \omega_0(s_{14}, t_{14}) + \left(\frac{\lambda_{23}}{\lambda}\right)^2 \omega_0(s_{23}, t_{23})\right)\right] \frac{m^6}{\mu^4}, \tag{116}$$

$$\omega(r) = \frac{1}{2\ln 2 - 1} \left[f(r) - \frac{1}{r^2}\right], \quad \omega_0(s, t) = \frac{1}{2\ln 2 - 1} \left[f(s) - \frac{1}{s^2} t^2 f(t)\right],$$

where the function f stands for the following combination of logarithms,

$$f(x) = \frac{1}{2}\left(1 + \frac{1}{x^2}\right)^2 \ln\left(1 + x^2\right) - \frac{1}{2}\left(1 - \frac{1}{x^2}\right)^2 \ln\left(1 - x^2\right), \tag{117}$$

and where the parameters s_{14}, t_{14}, s_{23}, and t_{23} are defined as follows (see Eq. (113)),

$$s_{14} = \frac{\lambda_{14}}{\sigma_{14}}, \quad t_{14} = \frac{\delta_{14}}{\sigma_{14}}, \quad s_{23} = \frac{\lambda_{14}}{\sigma_{23}}, \quad t_{23} = \frac{\delta_{14}}{\sigma_{23}}. \tag{118}$$

The weight function ω in Eq. (116) accounts for the relative importance of loop diagrams with internal M lines compared to loop diagrams with internal A lines. Similarly, ω_0 is a measure for the relative importance of loop diagrams involving neutral meson fields. It reduces to ω in the flavor-symmetric limit, which is characterized by the parameter t going to zero, i.e., $\omega_0(s, 0) = \omega(s)$. Both weight functions are normalized such that they smoothly interpolate between 0 and 1,

$$\omega(0) = 0, \quad \omega(1) = 1, \quad \omega_0(s, t = s) = 0, \quad \omega_0(1, 0) = 1. \tag{119}$$

Here, note that t can never exceed s by definition, $t \leq s$ (see Eqs. (113) and (118)). The two functions ω and ω_0 can, moreover, be conveniently approximated by the following polynomials,

$$\omega(r) \approx r^2, \quad \omega_0(s, t) \approx s^2 - \left(\frac{t}{s}\right)^2 t^2, \tag{120}$$

---

[16]Recall that, for the purposes of this appendix, we are assuming the Kähler potential to be canonical. However, in a more realistic context, we would also expect the presence of uncalculable higher-dimensional terms in the Kähler potential, which would yield further contributions to the sgoldstino mass (see our discussion related to Eq. (42)).

which nicely reproduce the exact identities in Eq. (119) as well as the fact that $\omega_0(s,0) = \omega(s)$.

Next, let us evaluate the sgoldstino mass in the flavor-symmetric limit, i.e., for all λ_0^a being equal to λ. This will provide us with a much simpler expression for m_{s_0} that approximates the full result in Eq. (116) reasonably well as long as there is no large hierarchy among the Yukawa couplings. In the flavor-symmetric limit, we are then allowed to perform the following simplifications,[17]

$$\delta_{14} = \delta_{23} = 0, \quad \lambda_{14} = \lambda_{23} = \sigma_{14} = \sigma_{23} = \lambda, \quad s_{14} = s_{23} = 1 \quad t_{14} = t_{23} = 0. \quad (121)$$

The weight function ω_0 therefore simply evaluates twice to unity, so that m_{s_0} turns into,

$$m_{s_0}^2 = \frac{2\ln 2 - 1}{16\pi^2} \left[1 + \omega(r) + \frac{4}{\rho^6}\right] \frac{m^6}{\mu^4} \approx \frac{2\ln 2 - 1}{16\pi^2} \left[1 + r^2 + \frac{4}{\rho^6}\right] \frac{m^6}{\mu^4}. \quad (122)$$

This is our final result for the sgoldstino mass. Plugging it into Eq. (105), we find for $\langle S_0 \rangle$

$$\langle S_0 \rangle = \frac{2\,(2-\zeta)^{3/2}}{(2\ln 2 - 1)\,[1 + \omega(r) + 4/\rho^6]\,\rho^6} \frac{16\pi^2}{\lambda^3}\, m_{3/2} \approx \frac{(2-\zeta)^{5/2}}{(2\ln 2 - 1)\,(4 - 2\zeta + \rho^6)} \frac{16\pi^2}{\lambda^3}\, m_{3/2}. \quad (123)$$

We stress that, allowing for the possibility of a noncanonical Kähler potential, this result receives corrections due to dynamically generated contributions to the sgoldstino mass. If we assume these contributions to be positive, the above expression for the goldstino VEV can also be understood as a conservative upper limit. On the other hand, for negative mass corrections coming from the dynamical Kähler potential, the actual value for $\langle S_0 \rangle$ ends up being larger than the expression in Eq. (123). In the main body of this paper, we shall assume that the former of these two possibilities is realized. In this case, working with our result in Eq. (123) will then correspond to a conservative treatment of the effect of higher-dimensional operators on the quality of the PQ symmetry.

Finally, we are ready to compute the VEVs of the singlet fields Z_+, Z_-, and X. Making use of the relations in Eq. (107) as well as of our result for $\langle S_0 \rangle$ in Eq. (123), we eventually find

$$\langle Z_\pm \rangle \approx \frac{(1-\zeta)^{1/2}\,(2-\zeta)^2}{(2\ln 2 - 1)\,(4 - 2\zeta + \rho^6)} \frac{16\pi^2}{\lambda^3}\, m_{3/2}, \quad \langle X \rangle \approx \frac{-\zeta^{1/2}\,(2-\zeta)^2}{(2\ln 2 - 1)\,(4 - 2\zeta + \rho^6)} \frac{16\pi^2}{\lambda^3}\, m_{3/2}. \quad (124)$$

It is instructive to consider the behavior of these expressions for certain extreme parameter choices. For $\zeta \to 0$, for instance, all three VEVs become arbitrarily large, $\langle Z_\pm \rangle, \langle |X| \rangle \gg m_{3/2}$. The reason for this is simply the inverse cubic power of λ appearing in all of the above VEVs. The two prefactors multiplying $16\pi^3/\lambda^3\, m_{3/2}$ in Eq. (124) stay, by contrast, also finite in the limit $\zeta \to 0$. For $\zeta \to 1$, on the other hand, $\langle Z_\pm \rangle$ approaches 0, while $\langle |X| \rangle$ is bounded from below,

$$\langle |X| \rangle \geq \frac{16\pi^2/\lambda_{\max}^3}{(2\ln 2 - 1)\,(2 + \rho^6)}\, m_{3/2} \simeq 0.07 \left(\frac{3}{2+\rho^6}\right) \left(\frac{4\pi}{\lambda_{\max}}\right)^3 m_{3/2}. \quad (125)$$

Interestingly enough, we also find that all three VEVs typically turn out to be smaller than $m_{3/2}$ for most values of ζ. More precisely, for $\kappa\,\eta = 4\pi$, $\rho = 1$ and in terms of the coupling λ, we obtain

$$\kappa\,\eta = 4\pi \quad \Rightarrow \quad \langle Z_\pm \rangle \leq m_{3/2} \text{ for } \lambda \gtrsim 2.0\,\pi, \quad \langle |X| \rangle \leq m_{3/2} \text{ for } \lambda \gtrsim 1.5\,\pi. \quad (126)$$

---

[17]We only perform these simplifications in order to obtain a more practical (approximate) expression for the sgoldstino mass, i.e., we do not assume that any larger global flavor symmetry is *actually* realized, since this would lead, for instance, to problems involving massless particles (see Eq. (112) as well as the discussion in Footnote 6).

This is advantageous from the perspective of our axion model, as it indicates that dangerous higher-dimensional operators involving powers of the fields Z_+, Z_- and/or X may not have as strong an effect on the quality of the PQ symmetry as one may naively expect (i.e., if one simply estimated $\langle Z_\pm \rangle$ and $\langle |X| \rangle$ to be some values of $\mathcal{O}(m_{3/2})$). Lastly, we mention that, in the limit of the deformed moduli constraint being exactly fulfilled, i.e., for $\kappa \to \infty$, the VEV of the singlet field X vanishes completely, $\langle |X| \rangle = 0$, while $\langle Z_\pm \rangle$ turns into a simple function of λ,

$$\kappa \to \infty \quad \Rightarrow \quad \langle Z_\pm \rangle = \frac{4}{(2 \ln 2 - 1)(4 + \rho^6)} \frac{16\pi^2}{\lambda^3} m_{3/2} \simeq 0.16 \left(\frac{5}{4 + \rho^6} \right) \left(\frac{4\pi}{\lambda} \right)^3 m_{3/2}. \quad (127)$$

Here, the fact that X vanishes is consistent with the observation that X becomes infinitely heavy for $\kappa \to \infty$, indicating that, in this limit, it is an unphysical field that needs to be integrated out.

References

[1] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. D **16**, 1791 (1977).

[2] For reviews, see: J. E. Kim, Phys. Rept. **150**, 1 (1987); J. E. Kim and G. Carosi, Rev. Mod. Phys. **82**, 557 (2010), arXiv:0807.3125 [hep-ph].

[3] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).

[4] J. Preskill, M. B. Wise and F. Wilczek, Phys. Lett. B **120**, 127 (1983); L. F. Abbott and P. Sikivie, Phys. Lett. B **120**, 133 (1983); M. Dine and W. Fischler, Phys. Lett. B **120**, 137 (1983).

[5] C. A. Baker, D. D. Doyle, P. Geltenbort, K. Green, M. G. D. van der Grinten, P. G. Harris, P. Iaydjiev and S. N. Ivanov *et al.*, Phys. Rev. Lett. **97**, 131801 (2006), hep-ex/0602020; Phys. Rev. Lett. **98** (2007) 149102, arXiv:0704.1354 [hep-ex].

[6] M. Kamionkowski and J. March-Russell, Phys. Lett. B **282**, 137 (1992) hep-th/9202003; S. Ghigna, M. Lusignoli and M. Roncadelli, Phys. Lett. B **283**, 278 (1992); B. A. Dobrescu, Phys. Rev. D **55**, 5826 (1997), hep-ph/9609221.

[7] R. Holman, S. D. H. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins and L. M. Widrow, Phys. Lett. B **282**, 132 (1992) hep-ph/9203206; S. M. Barr and D. Seckel, Phys. Rev. D **46**, 539 (1992).

[8] S. B. Giddings and A. Strominger, Nucl. Phys. B **307**, 854 (1988); S. R. Coleman, Nucl. Phys. B **310**, 643 (1988); G. Gilbert, Nucl. Phys. B **328**, 159 (1989); T. Banks and N. Seiberg, Phys. Rev. D **83**, 084019 (2011), arXiv:1011.5120 [hep-th].

[9] J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979); M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **166**, 493 (1980);

[10] A. R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 260 (1980) [Yad. Fiz. **31**, 497 (1980)]; M. Dine, W. Fischler and M. Srednicki, Phys. Lett. B **104**, 199 (1981).

[11] M. S. Turner, Phys. Rept. **197**, 67 (1990); G. G. Raffelt, Phys. Rept. **198**, 1 (1990).

[12] K. Choi and J. E. Kim, Phys. Lett. B **154**, 393 (1985), Erratum-ibid. **156B**, 452 (1985); P. Svrcek and E. Witten, JHEP **0606**, 051 (2006), hep-th/0605206.

[13] T. W. Donnelly, S. J. Freedman, R. S. Lytel, R. D. Peccei and M. Schwartz, Phys. Rev. D **18**, 1607 (1978).

[14] A. D. Linde, Phys. Lett. B **201**, 437 (1988); M. Tegmark, A. Aguirre, M. Rees and F. Wilczek, Phys. Rev. D **73**, 023505 (2006), astro-ph/0511774; B. Freivogel, JCAP **1003**, 021 (2010), arXiv:0810.0703 [hep-th].

[15] V. Barger, C. W. Chiang, J. Jiang and T. Li, Nucl. Phys. B **705**, 71 (2005), hep-ph/0410252; L. J. Hall, Y. Nomura and S. Shirai, JHEP **1406**, 137 (2014), arXiv:1403.8138 [hep-ph].

[16] B. Feldstein and T. T. Yanagida, Phys. Lett. B **720**, 166 (2013), arXiv:1210.7578 [hep-ph].

[17] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B **241**, 493 (1984); Phys. Lett. B **137**, 187 (1984); Nucl. Phys. B **256**, 557 (1985).

[18] M. Ibe, T. Moroi and T. T. Yanagida, Phys. Lett. B **644**, 355 (2007), hep-ph/0610277; M. Ibe and T. T. Yanagida, Phys. Lett. B **709**, 374 (2012), arXiv:1112.2462 [hep-ph]; N. Arkani-Hamed, A. Gupta, D. E. Kaplan, N. Weiner and T. Zorawski, arXiv:1212.6971 [hep-ph].

[19] M. Ibe, S. Matsumoto and T. T. Yanagida, Phys. Rev. D **85**, 095011 (2012), arXiv:1202.2253 [hep-ph];

[20] L. J. Hall and Y. Nomura, JHEP **1201**, 082 (2012), arXiv:1111.4519 [hep-ph]; A. Arvanitaki, N. Craig, S. Dimopoulos and G. Villadoro, JHEP **1302**, 126 (2013), arXiv:1210.0555 [hep-ph].

[21] L. J. Hall, Y. Nomura and S. Shirai, JHEP **1301**, 036 (2013), arXiv:1210.2395 [hep-ph].

[22] For a review, see: H. P. Nilles, Phys. Rept. **110**, 1 (1984).

[23] M. Dine and D. MacIntire, Phys. Rev. D **46**, 2594 (1992), hep-ph/9205227; G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP **9812**, 027 (1998), hep-ph/9810442; L. Randall and R. Sundrum, Nucl. Phys. B **557**, 79 (1999), hep-th/9810155; J. A. Bagger, T. Moroi and E. Poppitz, JHEP **0004**, 009 (2000), hep-th/9911029; P. Binetruy, M. K. Gaillard and B. D. Nelson, Nucl. Phys. B **604**, 32 (2001), hep-ph/0011081.

[24] F. D'Eramo, J. Thaler and Z. Thomas, JHEP **1206**, 151 (2012), arXiv:1202.1280 [hep-ph]; K. Harigaya and M. Ibe, Phys. Rev. D **90**, no. 8, 085028 (2014), arXiv:1409.5029 [hep-th].

[25] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012), arXiv:1207.7214 [hep-ex]; S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012), arXiv:1207.7235 [hep-ex].

[26] Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. **85**, 1 (1991); Phys. Lett. B **262**, 54 (1991); J. R. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B **257**, 83 (1991); H. E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991).

[27] M. Ibe, S. Matsumoto, S. Shirai and T. T. Yanagida, JHEP **1307**, 063 (2013), arXiv:1305.0084 [hep-ph].

[28] K. Harigaya, M. Ibe and T. T. Yanagida, JHEP **1312**, 016 (2013), arXiv:1310.0643 [hep-ph]; K. Harigaya, K. Kaneta and S. Matsumoto, Phys. Rev. D **89**, no. 11, 115021 (2014), arXiv:1403.0715 [hep-ph].

[29] J. L. Evans, M. Ibe, K. A. Olive and T. T. Yanagida, arXiv:1412.3403 [hep-ph].

[30] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross, Phys. Lett. B **131**, 59 (1983); M. Ibe, Y. Shinbara and T. T. Yanagida, Phys. Lett. B **639**, 534 (2006), hep-ph/0605252.

[31] S. Weinberg, Phys. Rev. Lett. **48**, 1303 (1982); J. R. Ellis, D. V. Nanopoulos and S. Sarkar, Nucl. Phys. B **259**, 175 (1985); M. Kawasaki, K. Kohri and T. Moroi, Phys. Lett. B **625**, 7 (2005), astro-ph/0402490; Phys. Rev. D **71**, 083502 (2005), astro-ph/0408426; K. Jedamzik, Phys. Rev. D **74**, 103509 (2006), hep-ph/0604251.

[32] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B **477**, 321 (1996), hep-ph/9604387; T. Moroi and M. Nagai, Phys. Lett. B **723**, 107 (2013), arXiv:1303.0668 [hep-ph]; D. McKeen, M. Pospelov and A. Ritz, Phys. Rev. D **87**, no. 11, 113002 (2013), arXiv:1303.1172 [hep-ph]; W. Altmannshofer, R. Harnik and J. Zupan, JHEP **1311**, 202 (2013), arXiv:1308.3653 [hep-ph]; M. Baumgart, D. Stolarski and T. Zorawski, Phys. Rev. D **90**, no. 5, 055001 (2014), arXiv:1403.6118 [hep-ph].

[33] G. Lazarides, C. Panagiotakopoulos and Q. Shafi, Phys. Rev. Lett. **56**, 432 (1986); M. Dine, In "Cincinnati 1992, Proceedings, Quantum gravity and beyond", 157-169 and Calif. Univ. Santa Cruz - SCIPP 92-027 (1992) 14 p, hep-th/9207045; A. G. Dias, V. Pleitez and M. D. Tonasse, Phys. Rev. D **69**, 015007 (2004), hep-ph/0210172; Phys. Rev. D **67**, 095008 (2003), hep-ph/0211107; K. S. Babu, I. Gogoladze and K. Wang, Phys. Lett. B **560**, 214 (2003), hep-ph/0212339; K. S. Choi, H. P. Nilles, S. Ramos-Sanchez and P. K. S. Vaudrevange, Phys. Lett. B **675**, 381 (2009), arXiv:0902.3070 [hep-th]; L. M. Carpenter, M. Dine and G. Festuccia, Phys. Rev. D **80**, 125017 (2009), arXiv:0906.1273 [hep-th].

[34] K. Harigaya, M. Ibe, K. Schmitz and T. T. Yanagida, Phys. Rev. D **88**, no. 7, 075022 (2013), arXiv:1308.1227 [hep-ph].

[35] Y. Imamura, T. Watari and T. Yanagida, Phys. Rev. D **64**, 065023 (2001), hep-ph/0103251; R. Kappl, B. Petersen, S. Raby, M. Ratz, R. Schieren and P. K. S. Vaudrevange, Nucl. Phys. B **847**, 325 (2011), arXiv:1012.4574 [hep-th]; M. Dine and A. Monteux, JHEP **1401**, 011 (2014), arXiv:1212.4371 [hep-ph].

[36] N. Sakai and T. Yanagida, Nucl. Phys. B **197**, 533 (1982); S. Weinberg, Phys. Rev. D **26**, 287 (1982).

[37] K. I. Izawa and T. Yanagida, Prog. Theor. Phys. **97**, 913 (1997), hep-ph/9703350.

[38] A. E. Nelson and N. Seiberg, Nucl. Phys. B **416**, 46 (1994), hep-ph/9309299.

[39] K. A. Intriligator, N. Seiberg and D. Shih, JHEP **0604**, 021 (2006), hep-th/0602239.

[40] J. E. Kim and H. P. Nilles, Phys. Lett. B **138**, 150 (1984).

[41] For reviews, see: M. Maniatis, Int. J. Mod. Phys. A **25**, 3505 (2010), arXiv:0906.0777 [hep-ph]; U. Ellwanger, C. Hugonie and A. M. Teixeira, Phys. Rept. **496**, 1 (2010), arXiv:0910.1785 [hep-ph].

[42] K. Inoue, M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. D **45**, 328 (1992); J. A. Casas and C. Munoz, Phys. Lett. B **306**, 288 (1993), hep-ph/9302227; see also: G. F. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988).

[43] E. Dudas, Y. Mambrini, A. Mustafayev and K. A. Olive, Eur. Phys. J. C **72**, 2138 (2012), Erratum-ibid. C **73**, 2430 (2013), arXiv:1205.5988 [hep-ph].

[44] J. L. Evans, M. Ibe, K. A. Olive and T. T. Yanagida, Eur. Phys. J. C **73**, 2468 (2013), arXiv:1302.5346 [hep-ph]; Eur. Phys. J. C **73**, no. 10, 2611 (2013), arXiv:1305.7461 [hep-ph].

[45] L. M. Krauss and F. Wilczek, Phys. Rev. Lett. **62**, 1221 (1989); T. Banks and M. Dine, Phys. Rev. D **45**, 1424 (1992), hep-th/9109045.

[46] L. E. Ibanez and G. G. Ross, Phys. Lett. B **260**, 291 (1991); Nucl. Phys. B **368**, 3 (1992); L. E. Ibanez, Nucl. Phys. B **398**, 301 (1993), hep-ph/9210211.

[47] K. Kurosawa, N. Maru and T. Yanagida, Phys. Lett. B **512**, 203 (2001), hep-ph/0105136.

[48] T. Yanagida, in Proceedings of the *"Workshop on the Unified Theory and the Baryon Number in the Universe"*, Tsukuba, Japan, Feb. 13-14 (1979), edited by O. Sawada and A. Sugamoto, KEK report KEK-79-18, Conf. Proc. C **7902131**, 95 (1979); Prog. Theor. Phys. **64**, 1103 (1980); M. Gell-Mann, P. Ramond and R. Slansky, in *"Supergravity"*, North-Holland, Amsterdam (1979), edited by D. Z. Freedom and P. van Nieuwenhuizen, CERN report Print-80-0576, Conf. Proc. C **790927**, 315 (1979), arXiv:1306.4669 [hep-th]; see also: P. Minkowski, Phys. Lett. B **67**, 421 (1977).

[49] J. L. Evans, M. Ibe, J. Kehayias and T. T. Yanagida, Phys. Rev. Lett. **109**, 181801 (2012), arXiv:1111.2481 [hep-ph].

[50] M. Dine and J. Kehayias, Phys. Rev. D **82**, 055014 (2010), arXiv:0909.1615 [hep-ph].

[51] K. I. Izawa and T. Yanagida, Prog. Theor. Phys. **95**, 829 (1996), hep-th/9602180; K. A. Intriligator and S. D. Thomas, Nucl. Phys. B **473**, 121 (1996), hep-th/9603158.

[52] M. B. Green and J. H. Schwarz, Phys. Lett. B **149**, 117 (1984).

[53] H. M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg and P. K. S. Vaudrevange, Phys. Lett. B **694**, 491 (2011), arXiv:1009.0905 [hep-ph]; Nucl. Phys. B **850**, 1 (2011), arXiv:1102.3595 [hep-ph].

[54] K. Nakayama and T. T. Yanagida, Phys. Lett. B **722**, 107 (2013), arXiv:1302.3332 [hep-ph]; J. L. Evans and K. A. Olive, Phys. Rev. D **90**, no. 11, 115020 (2014), arXiv:1408.5102 [hep-ph].

[55] A. Manohar and H. Georgi, Nucl. Phys. B **234**, 189 (1984); H. Georgi and L. Randall, Nucl. Phys. B **276**, 241 (1986); M. A. Luty, Phys. Rev. D **57**, 1531 (1998), hep-ph/9706235; A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Lett. B **412**, 301 (1997), hep-ph/9706275.

[56] N. Seiberg, Phys. Rev. D **49**, 6857 (1994), hep-th/9402044.

[57] L. O'Raifeartaigh, Nucl. Phys. B **96**, 331 (1975).

[58] V. Domcke, K. Schmitz and T. T. Yanagida, Nucl. Phys. B **891**, 230 (2015), arXiv:1410.4641 [hep-th].

[59] K. Tamvakis and D. Wyler, Phys. Lett. B **112**, 451 (1982); K. Rajagopal, M. S. Turner and F. Wilczek, Nucl. Phys. B **358**, 447 (1991).

[60] For a review, see: M. Kawasaki and K. Nakayama, Ann. Rev. Nucl. Part. Sci. **63**, 69 (2013), arXiv:1301.1123 [hep-ph].

[61] T. Kugo, I. Ojima and T. Yanagida, Phys. Lett. B **135**, 402 (1984).

[62] Z. Chacko, M. A. Luty and E. Ponton, JHEP **9812** (1998) 016, hep-th/9810253.

[63] G. Veneziano and S. Yankielowicz, Phys. Lett. B **113** (1982) 231; V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **229**, 407 (1983); E. Cohen and C. Gomez, Phys. Rev. Lett. **52**, 237 (1984).

[64] S. P. Martin, Phys. Rev. D **46**, 2769 (1992), hep-ph/9207218; Phys. Rev. D **54**, 2340 (1996), hep-ph/9602349.

[65] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).

[66] B. Bhattacherjee, B. Feldstein, M. Ibe, S. Matsumoto and T. T. Yanagida, Phys. Rev. D **87**, no. 1, 015028 (2013), arXiv:1207.5453 [hep-ph].

[67] P. Sikivie, Phys. Rev. Lett. **48**, 1156 (1982).

[68] M. S. Turner, Phys. Rev. D **33**, 889 (1986).

[69] A. Arvanitaki and A. A. Geraci, Phys. Rev. Lett. **113**, no. 16, 161801 (2014), arXiv:1403.1290 [hep-ph].

[70] V. Anastassopoulos, S. Andrianov, R. Baartman, M. Bai, S. Baessler, J. Benante, M. Berz and M. Blaskiewicz *et al.*, arXiv:1502.04317 [physics.acc-ph].